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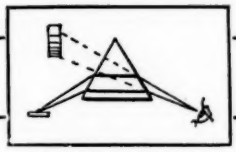

JANUARY
1935

CONTENTS

Notes on Resistances of Selenium Cells Deposited in Certain Gases . . . . . LYNN W. JONES 1

Effect of Temperature upon Reciprocity Law Failure in Photographic Exposure . . . . . J. H. WEBB 4

Maxwell Triangle Yielding Uniform Chromaticity Scales . DEANE B. JUDD 24

Photometry and Energy Distribution of the High Intensity Mercury Lamp . . . . . L. B. JOHNSON 36

Study, by Means of Huygen's Principle, of the Reflection of a Spherical Light Wave from a Moving Plane Mirror . . NADIASHDA GALLI-SHOHAT 39

Proceedings of the Nineteenth Annual Meeting of the Optical Society of America, Washington, D. C., October 18–20, 1934 42

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Journal of the
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Notes on Resistances of Selenium Cells Deposited in Certain Gases

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(Received October 27, 1934)

THE work of Datta[1] on the variation of resistance of selenium cells with temperature and the measurements of Tisdale[2] on the effect of gases on the electrical properties of selenium have suggested the measurement of variations of resistance of selenium cells deposited at different temperatures in oxygen and carbon dioxide. Certain precautions relative to contacts and preservation of cells which become apparent from the historical survey of the early investigations on such cells are considered in the construction of the cells made in this investigation.

The cells which were used were of the Lindner-Replogle[3] type and were made by thinly depositing platinum paint on small glass plates. A two stage heating process was used to secure the zigzag design and to form the very hard surface which is necessary for the soldering of the connecting leads. The preliminary heating at 182 degrees C left a soft, black surface which was easily worked and the second heating at 550 degrees C completed the process. The selenium was deposited upon the plates with the well-known distillation process. The plates were held in position with an adjustable telescopic cylinder, the height chosen depending upon the depositing temperature desired for each cell, this temperature being a function of the gradient of the vapor stream above the boiling selenium source. The depositing temperatures were measured with a calibrated thermocouple. The commercial gases used were dried before passing into the bell jar which surrounded the distilling apparatus. After formation the leads were soldered on the cells and the selenium deposit covered with calcium chloride desiccating caps which were attached with sealing wax. The cell resistances were measured with a calibrated Wheatstone's bridge and a standard lamp. The light from this standard source struck the gridded side of the cells so that for a given intensity of illumination the thickness of deposit beyond a minimum thickness had no effect upon the resistance of the cell. Experimentation with several cells deposited at the same temperature indicated that the time of deposit beyond a minimum of one hour had little effect upon the resistance of the cell. (See Fig. 1.)

Denoting the resistance of each cell when exposed to a constant source of illumination by the term "light" and the resistance of the cell when covered by the term "dark" Fig. 2 was plotted to show the relation between the depositing temperatures of nineteen cells formed in oxygen and their "light" and "dark" resistances. The term "depositing temperature" as applied to each cell refers to the particular temperature at

[1] S. Datta, Phil. Mag. [6] **42**, 463–470 (1921).
[2] W. E. Tisdale, Phys. Rev. **12**, 325–335 (1918).
[3] O. Lindner and J. B. Replogle, El. World **59**, 251 (1912).

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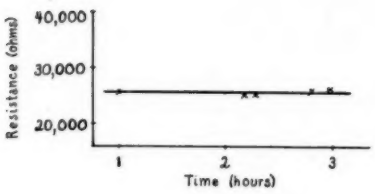

FIG. 1. "Dark" resistance-depositing time curve. Showing the relation between "dark" resistance and the depositing time of five cells formed at the same temperature.

assumed by the slight rise in his curve that there is an increase of resistance with increases of temperature above 165°C. The curves of Figs. 2 and 3 actually show the nature of this increase. Fig. 5 shows the variation of conductance of two sample cells with time in days, and a similar conductance-time curve secured by Ries[4] which was taken with an uncapped, undesiccated cell. The ordinate value of Ries' curve is reproduced

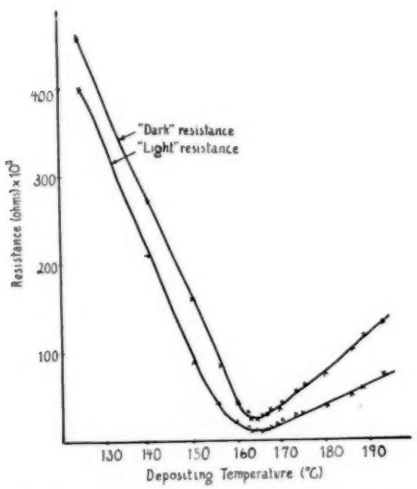

FIG. 2. Resistance-depositing temperature curves for oxygen which show the relation between various temperatures at which the cells were formed and the "light" and "dark" resistances.

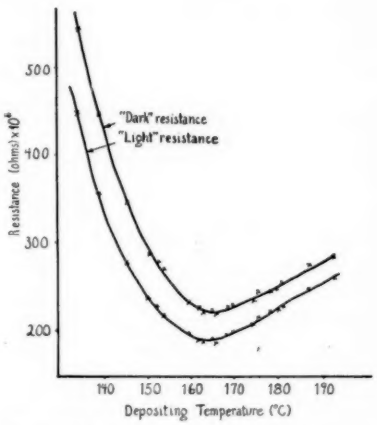

FIG. 3. Resistance-depositing temperature curves for carbon dioxide which show the relation between various temperatures at which the cells were formed and the "light" and "dark" resistances.

which that cell was formed. For example, one cell was formed at a depositing temperature of 180°C and its "light" and "dark" resistances measured immediately after preservation. Other cells were formed at temperatures ranging from 125 to 193°C. Fig. 3 shows similar curves which were secured by depositing twenty cells in carbon dioxide. The general forms of these curves agree quite well with that of Datta (Fig. 4) which he secured while investigating the variation of resistance of selenium with temperature throughout the range of 0 to 170°C. There appears to be a region of minimum resistance at approximately 160°C on Datta's curve which also agrees with the region of minimum resistance at 164°C as shown in Figs. 2 and 3. Since the curve of Datta does not extend beyond 165°C it can only be

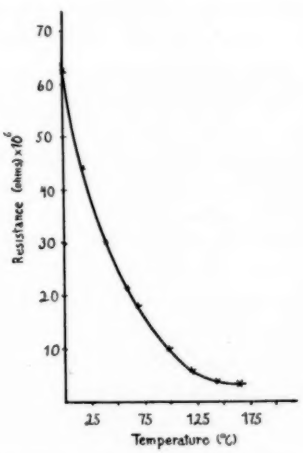

FIG. 4. Resistance-temperature curve of Datta showing the variation of selenium resistance with temperature.

[4] C. Ries, Phys. Zeits. 9, 569–582 (1908).

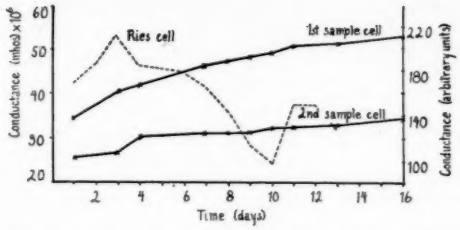

FIG. 5. Conductance-time curve illustrating variations of conductance with time of two sample cells. The graph also shows the conductance (in arbitrary units) and time curve secured by Ries.

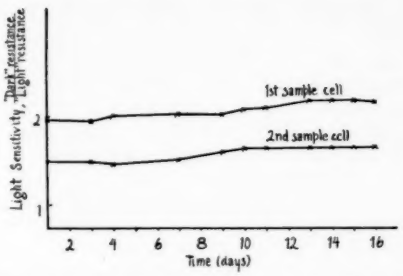

FIG. 6. Light sensitivity-time curve showing a slight increase in the sensitivity of two sample cells over a period of sixteen days.

on the right-hand side of Fig. 5. The conductance properties of Ries' cell fell off during a period of 13 days, with certain large fluctuations entering in, until the conductance was only two-thirds of the original value. The sample cells show continuous rises in conductance. These results indicate the advantages secured in preserving the cells with desiccated caps. The sensitivity of a cell is given by the ratio of "dark" to "light" resistance. There is a slight increase in the sensitivity of the two sample cells over a period of several days as shown in Fig. 6. This increase seems to indicate that the desiccating caps improve the sensitivity of the cells.

CONCLUSIONS

The results secured from this investigation indicate that when selenium cells are deposited in oxygen and carbon dioxide the variations of resistance with depositing temperature are similar to the variations of resistance with temperature of selenium. It was also found that the resistances of cells deposited in carbon dioxide are very much higher than those deposited in oxygen. This fact agrees with the conclusions of Tisdale that "the gases of heavier molecular weight appear to have the most detrimental effects." Further, that a minimum region of resistance, characteristic of each gas, is found at a temperature of about 164°C, which is probably, according to the reasoning of Marc,[5] the temperature at which the most stable form of selenium is secured through a distillation and depositing process. Above this temperature there must be a change within the crystal structure of the selenium and below this temperature there is a change from the stable selenium crystals into the labile and nonconducting forms *A* and *B*.

It was also found that, by using a hard platinum grid, cells could be secured which had very low contact resistances. The terminals could be soldered to these hard grids and the older forms of high resistance, pressure contacts were eliminated.

Selenium cells may also have their conductance and sensitivity properties preserved by the application of calcium chloride drying caps.

ACKNOWLEDGMENTS

The author wishes to thank Doctor Loughridge of the University of Washington Department of Physics for his many helpful suggestions relative to this problem, and to express his appreciation to Doctor Vollrath of the University of Southern California Department of Physics for his assistance in the interpretations of this experiment.

[5] R. Marc, D. Chem. Ges. Ber. **39**, 697–704 (1906).

The Effect of Temperature upon Reciprocity Law Failure in Photographic Exposure*

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(Received October 11, 1934)

Measurements of the effect of temperature variation on the reciprocity failure curves were made for several types of emulsions and for radiation of several different wavelengths. The effect of temperature variation upon emulsion sensitivity is shown to be very strongly dependent upon the intensity level at which the exposures are made. At very low intensities it is found that the speeds of certain emulsions can be increased several fold by lowering the temperature. An analysis of the data suggests a possible manner of division of the reciprocity curves into two component parts which, presumably, correspond to two different modes of utilization of the exposure in the formation of the latent image. A theoretical formulation based on the foregoing analysis yields the Kron-Halm catenary equation for representing the reciprocity failure curves.

INTRODUCTION

THE present experimental work had its origin in connection with some work on the intermittency effect carried out by the author some months ago. In order to test at that time an hypothesis with regard to the intermittency effect, it was necessary to make some exposures at low temperature. These exposures were made at very low light intensities and the results of the experiments indicated a slight increase of emulsion sensitivity at the lowered temperature, which was approximately that of solid carbon dioxide. It was decided to compare this result with the results of similar exposures made at high intensity, and further experiments were accordingly performed. These latter exposures were made at intensities of the order used in ordinary photographic exposure, and the results showed that the lowered temperature produced a decrease in emulsion sensitivity, the exact reverse of the previous results.

This difference in the behavior of emulsion sensitivity with temperature, at different levels of intensity, suggested at once that the reciprocity law failure curves, which portray emulsion sensitivity as a function of the intensity of exposing radiation, should be altered in shape by temperature variation. A few preliminary experiments verified this conclusion, and the results of these experiments showed such a pronounced effect upon the reciprocity characteristic that it was decided to make a thorough investigation of this subject.

Before proceeding to a description of this work, a brief review of the literature on the general subject of variation of emulsion sensitivity with temperature seems appropriate. This brief résumé of the literature will show that the observation made above with regard to the part played by the intensity is not new, but has been noted by others. However, the effect has never been studied systematically over a wide range of intensities, nor from the standpoint of the alteration in the reciprocity failure characteristic. It is this last feature, relative to the effect of temperature upon reciprocity law failure, that is considered of chief interest in such measurements and that renders them a source of information relative to the mechanism of reciprocity law failure and latent image formation.

Measurements on the effect of temperature upon emulsion sensitivity date back to 1884 and have been carried out by a number of investigators. Considerable discordance appears among the results obtained on this effect by different experimenters. For the most part, the workers have reported that an increase in temperature is accompanied by an increase in sensitivity, though there are exceptions to this. Abney[1] found that heating a photographic plate with a hot iron increased the sensitivity many-fold, while cooling with ice decreased the sensitivity. A. and L. Lumière[2] found that exposures 300 to 400 times as great were required at liquid air temperature as at room temperature. Precht and Schellen[3]

* Communication No. 538.

[1] W. DeW. Abney, Phot. News **28**, 315 (1884).
[2] A. and L. Lumière, Comptes rendus **128**, 359 (1899).
[3] J. Precht and A. Schellen, Arch. wiss. Phot. **1**, 58 (1899).

working over the range of temperature −32°C to 90°C found a very small variation in sensitivity. Measurements of Dalezki[4] showed that there is an increase in sensitivity with temperature over the range 7°C to 64°C, the temperature coefficient being of the order 1.05. Zimmern[5] found that the sensitivity of an emulsion increased with temperature over the range from room temperature to 80°C but that the increase was small.

Later, E. S. King[6] of Harvard College Observatory measured the effect of temperature upon emulsion sensitivity for intensities comparable with those used in stellar photography. King states in his book on *Stellar Photography*, page 56, "A change from 70°F to 0°F has been known to increase the sensitiveness by half a magnitude, a gain of about fifty percent, but different emulsions may vary in this respect." This is the first mention, known to the writer, of the fact that emulsion sensitivity can be increased by lowering the temperature, and this result is in apparent disagreement with the findings of all previous workers.

In 1908, Wallace[7] carried out some measurements on the effect of temperature upon sensitivity in an effort to determine the reason for the discordance between the results of King and the results of workers previous to King. Wallace's experiments were made under controlled intensity conditions and he succeeded in showing how the results of both King and his predecessors might be brought into accord. Wallace employed intensity scale H. & D. curves in his experiments, and accordingly, his intensity varied over a wide range of values in a single exposure strip. He found that the gradation of such intensity scale curves was altered considerably by temperature change. Choosing the curve taken at 24°C for reference, he noted that curves taken at temperatures between 24°C and 75°C crossed the one obtained at 24°C. From this result it was clear that at intensities below the point where the curves crossed, an increase in temperature resulted in a decrease in speed, while at intensities above the crossing point an increase in temperature resulted in an increase in speed. This finding allowed an explanation of the apparently contradictory results of King and his predecessors, as King worked definitely at low values of intensity while his predecessors had worked at moderate or high values of intensity. This appears to be the first definite statement of the fact that intensity plays a part in the effect which temperature has upon emulsion sensitivity, and it seems to have been overlooked by later workers who apparently have not considered the part played by intensity in such measurements.

Probably the most complete experiments that have been carried out on the effect of temperature upon sensitivity were the recent experiments by Eggert and Luft,[8] although this investigation did not include a study of the effect of intensity. These workers made measurements over the range of temperatures −60°C to 90°C, and extended their experiments to the cases of x-rays and alpha-particles. They found that the sensitivity of an emulsion to ordinary light first increased with temperature, passed through a maximum, and then decreased. This maximum sensitivity to light was found to occur in the general region of −20°C to −40°C under the conditions of their experiments. At very high temperatures (80°C), the sensitivity again started to rise, but fogging quickly set in to prevent further variation. The behavior in the cases of exposures to x-rays and alpha-particles was found to be markedly different from that in the case of light. The sensitivity to x-rays increased uniformly with temperature over the complete temperature range covered, while the sensitivity to alpha-particles was found to be independent of temperature variation.

The most recent measurements of the effect of temperature upon emulsion sensitivity were those by Sheppard, Wightman and Quirk,[9] of this laboratory. These measurements, covering a

[4] G. Dalezki, Zeits. f. wiss. Phot. **18**, 233 (1919).
[5] A. Zimmern, Comptes rendus **174**, 453 (1922).
[6] E. S. King, Photo-Beacon **17**, 267 (1905).
[7] R. J. Wallace, Astrophys. J. **28**, 39 (1908).
[8] J. Eggert and F. Luft, Veröffent. wiss. Zentral-Lab. Phot. Agfa **2**, 9 (1931).
[9] S. E. Sheppard, E. P. Wightman and R. F. Quirk, J. Phys. Chem. **38**, 817 (1934).

range of temperature of −180°C to 20°C, were made in much the same way as the experiments carried out by Wallace. The exposures consisted of intensity scale curves obtained by imaging a step tablet on the sensitive emulsion, which was maintained at definite temperatures. The results obtained in these experiments showed that gamma was markedly affected by temperature variation. Thus, indirectly the influence of the intensity level upon the effect of temperature was demonstrated. These results are, insofar as comparison can be made, in complete agreement with the results of the present work.

The investigation to be reported here consists of measurements of the effect of temperature upon the reciprocity failure characteristic for three different types of emulsion. The measurements cover a temperature range of −75°C to 50°C, and an intensity range of 1 to 10,000. Measurements on three different types of emulsion were made with monochromatic light of wavelength 4360A. A study of the effect of temperature at different wavelengths was made by exposing a single emulsion to radiations of wavelengths 3650A, 4360A and 5460A.

The results obtained in this work have shown that the effect of temperature upon emulsion sensitivity is so intimately connected with the intensity level at which the exposures are made that such measurements have little significance unless the intensity of the exposures is specified.

An analysis of the reciprocity failure curves at different temperatures has led to an interpretation which is believed to yield some information concerning the mechanism of reciprocity law failure and thereby the process involved in latent image formation.

The results found here are thought to possess some practical value in that they show the effects to be expected in ordinary pictorial photography under extreme conditions of temperature, such as those experienced in aerial work and in extreme geographical locations. Also, certain types of emulsion, at low intensities, are found to exhibit an increase of speed of from four to eight times with lowered temperature, a fact which should be of some interest to spectroscopists and astronomers.

Apparatus and Experimental Procedure

Sensitometer

The exposures in these experiments were made on film placed in a controlled temperature chamber which was mounted directly behind the sector of the reciprocity sensitometer. This reciprocity sensitometer is a time scale, logarithmic sector type of instrument and its design permits a large variation of exposure times. By a system of gear changes, the time of rotation of the sector can be varied in steps of powers of two from 0.0625 second to 72 hours, 49 minutes and 4 seconds. As a description of this instrument has been given in a paper by L. A. Jones,[10] the reader is referred to this paper for further details concerning the design of the instrument.

Temperature control apparatus

The constant temperature chamber in which the film was placed for exposure is shown in Fig. 1. As may be seen, it consists of a brass, double-walled container made in the form of a horizontal cylinder. This cylinder has a horizontal rectangular slot (10 by 1 cm) in one side through which the exposing radiation was allowed to pass to the film inside. Over this slot was placed a double-walled optical glass window, which could be exhausted during the low temperature work. The film for the experiment was placed on an inner cylindrical drum which could be slipped in and out of the outer double-walled vessel. This film drum has flanges at the ends to protect the film from damage during insertion in the apparatus. An axle passes through this drum and the end of the axle toward the inside rests in a half-bearing support. The other end of the axle extends through the apparatus to the outside and has a handle attached for rotating the film into different positions inside the apparatus. The large circular disk of brass between the drum and handle fits snugly into the end of the outer double-walled cylinder, and is held in position by means of two slots and pins on the periphery. As may be seen from Fig. 1, the inner drum upon which the film was wrapped has a shutter incorporated on one side which when turned to a position blocking the exposure window, makes

[10] L. A. Jones, J. Opt. Soc. Am. **7**, 305 (1923).

FIG. 1. Apparatus for controlling the temperature of photographic film during exposure.

the apparatus light-tight. The position of the inner drum and film can be accurately set by the index circle under the handle on the outside of the apparatus. The size of the film drum made it possible to obtain a maximum of twelve exposure curves on a single wrap of film. This was a necessary feature at low temperatures since it would have been impracticable, on account of condensation, to change film during a run. The placing of all exposures of a single run on one piece of film, and the subsequent development of all these exposures together, had the additional advantage of eliminating to a large extent the errors arising from small irregularities in emulsion sensitivity, coating thickness, and development.

For maintaining the temperature of the apparatus at any fixed value, a suitable bath was used in the outer double-walled cylinder. This double-walled space extended completely around the apparatus except for the exposing window and the end of the apparatus through which the film drum was removed. Over this end, a piece of cork one inch thick was cemented to the inside of the removable disk to make it practically impervious to heat. Over the entire apparatus was placed a layer of cotton approximately one inch thick, and this was held in place by a layer of quarter-inch white felt.

The various means for obtaining the different temperatures employed in this work are given below:

Temperature	Bath mixture
50°C	Water, heated by an electric heater
20°C	Water, at approximately room temperature
0°C	Water and crushed ice
−10, −14°C	Crushed ice and NaCl
−40°C	Alcohol, cooled by a lump of solid CO_2 at intervals
−75°C	Alcohol and crushed solid CO_2

The −40°C point was maintained with a surprising degree of precision considering the manner of control. The temperature of the bath, as measured by a pentane thermometer, varied between −42°C and −38°C. However, it is believed that the heat capacity of the apparatus was sufficient to decrease this variation to a very small amount at the film. This is shown by the smoothness of the data of all the curves obtained at the −40°C point.

In order to determine how closely the temperature of the film approached that of the bath, and the length of time required to reach a steady state of temperature within the apparatus, preliminary experiments were performed in which a thermocouple arrangement was used. For this

determination, one junction of the thermocouple was placed in contact with the film in front of the exposing window, while the other junction was placed in the bath mixture itself. When the bath liquid of lowest temperature (−78°C) was used, the thermocouple indicated a temperature at the film of −75°C. Equilibrium was reached in less than one hour. To check the constancy of the temperature of the bath liquids during the routine exposures, a pentane thermometer was used.

Light source

The light source employed was a horizontal type, quartz, mercury arc, operated at 60 volts and 2.6 amperes. The arc was enclosed in a blackened metal lamphouse containing a round window of approximately 1.5 inches in diameter, directly behind which the arc was mounted. Several pieces of ground glass were placed over the window to act as a diffusing medium. In front of the window were placed the filters for segregating the different wavelength radiations from the arc. The different filters employed for this purpose along with the line transmitted by each are listed below:

Radiation	Filter
3650A	Wratten No. 18A+1 cm. 25% $CuSO_4$
4360A	Corning No. 7.1 (Novial A Shade+Blue Purple Ultra, 4 mm)
5460A	Wratten No. 77A+No. 58

Measurement of the absolute value of intensity (ergs/cm²/sec.) from the lamp, for the 4360A line, was made by comparing the density obtained in an exposure to this light source, with the densities obtained on a calibrated plate. The energy in absolute units required to produce a known density upon this calibrated plate was determined on a special monochromatic sensitometer with a thermopile incorporated for energy measurements. The thermopile used was calibrated by means of a Hefner Amyl Acetate lamp according to a method given by Coblentz.[11]

The intensities of the lines of the mercury arc, obtained with the other filters, were found by reference to the relative spectral energy curve of Fig. 2, which applies to one of the emulsions used in these experiments. In deriving these intensity values, the reciprocity law failure was taken into account in cases where the time of exposure to the lamp intensity was different from the exposure time of the spectral energy curve. High precision is not claimed for these energy measurements, although they are believed to have an error of less than 10 percent, which is sufficiently good for the use to which they are put.

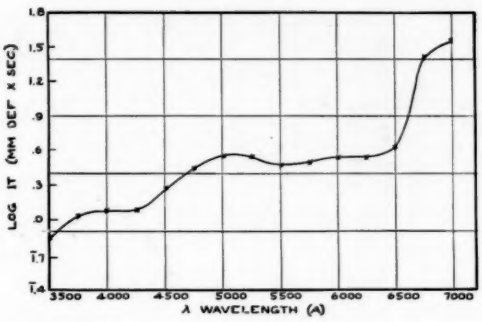

FIG. 2. Relative spectral energy curve of emulsion I, for density 1.0.

The variation in intensity for different points on a reciprocity curve was obtained by varying the distance from lamp source to film by moving the lamp along a track, thus varying the intensity according to the inverse square law. With the automatic voltage control used on the arc, and this method of varying intensities, the relative values of intensity have a precision of the order of plus or minus three percent.

Exposures

After placing a piece of film on the film drum in the temperature control apparatus, an exposure was first made at room temperature (20°C). This was done in order to have on each experimental film a control curve by means of which the effect of temperature upon the emulsion before and after exposure could be determined. This procedure also gave a reference point by means of which the reciprocity curve at any temperature could be properly placed on the reciprocity diagram relative to the 20°C curve. The shifts required here, because of slight differences in development from one run to another, were always small. Also, it should be added here that no consistent effect resulting

[11] W. W. Coblentz, Bull. Bur. Standards **11**, 87 (1915).

from the effect of temperature before or after exposure was noticed, except in cases of the low intensity exposure at high temperatures where some evidence of growth of the image appeared. After this preliminary exposure, the temperature control apparatus was brought to the temperature at which it was desired to determine the reciprocity curve by placing the proper bath mixture in the apparatus. A full hour was then allowed to elapse before making exposures in order to permit the apparatus to reach an equilibrium state of temperature. The lamphouse was then placed at a position such that its window was one meter from the film to be exposed, and the time of rotation of the sector was set to give a maximum exposure time, usually one second, suitable for this intensity. The film was next rotated to the number one position by means of the handle on the temperature control apparatus, and the sensitometer sector allowed to make one revolution for the exposure. For the succeeding exposure, the lamp was moved to a position double its former distance, thus quartering the intensity, and the time of rotation of the sector was correspondingly altered to four times its former value. This procedure was continued until exposures of approximately one hour's duration had been reached.

Development

After obtaining the complete set of exposures as outlined above for a single reciprocity failure curve, the film was removed from the temperature control apparatus. If the temperature of the apparatus had been below room temperature during the experiment, the film was not removed until the apparatus had warmed up to room temperature. Development was carried out in the standard Eastman D-90 developer, for a period of four minutes, at 68°F. During development, the film was lightly brushed with a camel's hair brush to insure uniformity of development conditions over the entire area of exposed film.

Densities

The densities were read on a Martens polarization photometer type of visual densitometer.

Reduction of data

The data obtained from the original exposures were plotted in the form of H. & D. curves, density as a function of log exposure, one curve being obtained for each intensity level used. The exposure value required to produce unit density was then ascertained from these H. & D. curves. This procedure gave exposure values, for unit density, as a function of intensity. The logarithms of these exposure values were then plotted as a function of the logarithm of intensity to give the characteristic reciprocity law failure curve. Lines of unit slope on the (log IT, log I) reciprocity diagram are lines of constant time, and their function is to designate the exposure times for those points at which they intersect the reciprocity curve. These lines have been placed on all the reciprocity diagrams in order to facilitate ready interpretation of these curves in terms of exposure times.

Results

The types of photographic materials used in these experiments are listed below. They have been assigned numbers for reference, and hereafter will be referred to by number only.

I—High-speed, panchromatic, negative emulsion

II—High-speed, blue-sensitive, negative emulsion

III—Medium-speed, blue-sensitive, negative emulsion

Effect of temperature upon the reciprocity characteristic

The reciprocity curves given in Figs. 3, 4 and 5 are for emulsions I, II and III, respectively. These curves illustrate, for each of the emulsions tested, the manner in which the reciprocity characteristic is altered with change of temperature. These curves were obtained with radiation of wavelength 4360A, and pertain to the density value stated on the figure.

An inspection of the curves of Figs. 3, 4 and 5 shows, for each of the three types of emulsion, that there is a marked change in the reciprocity characteristic with temperature. Disregarding

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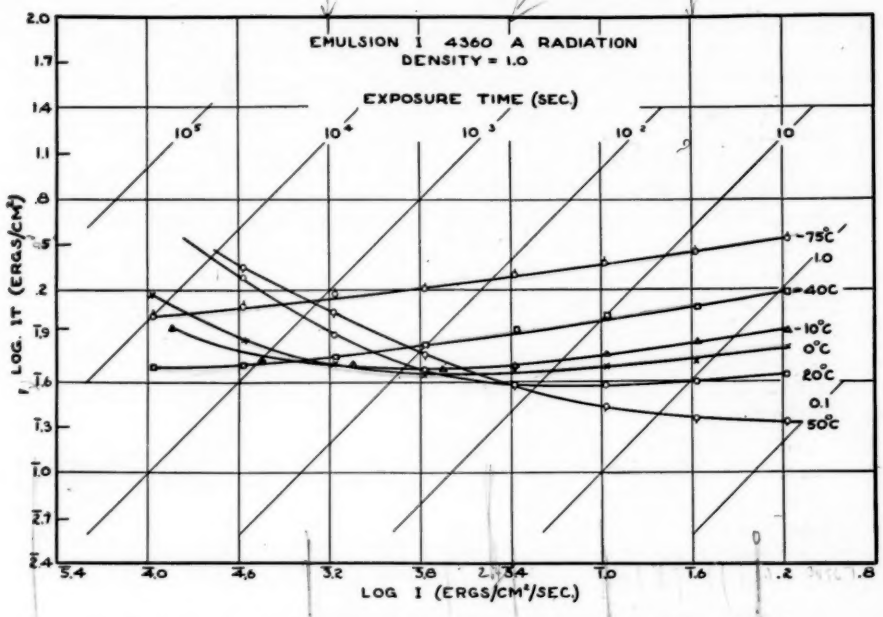

FIG. 3. Variation of the reciprocity characteristic with temperature for emulsion I.

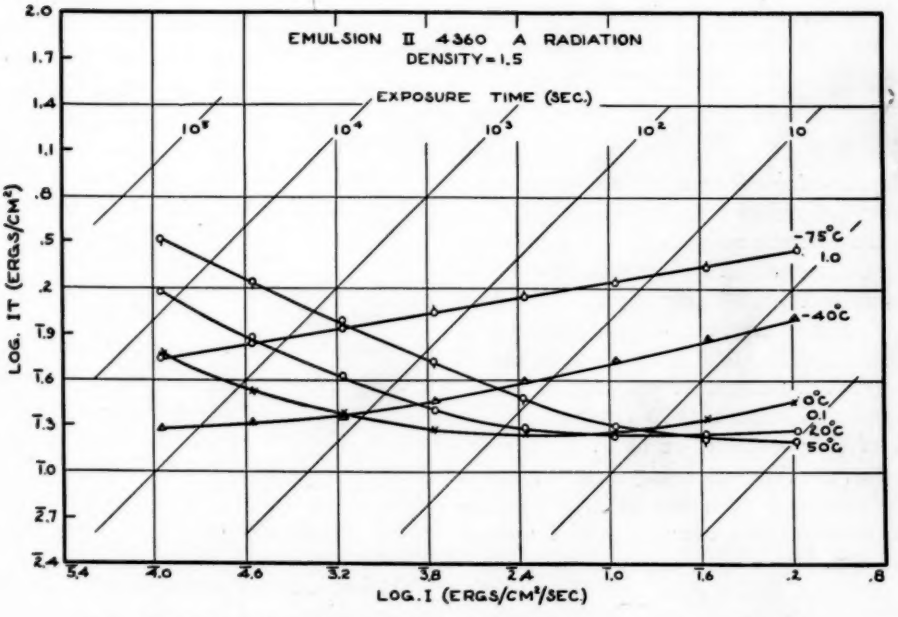

FIG. 4. Variation of the reciprocity characteristic with temperature for emulsion II.

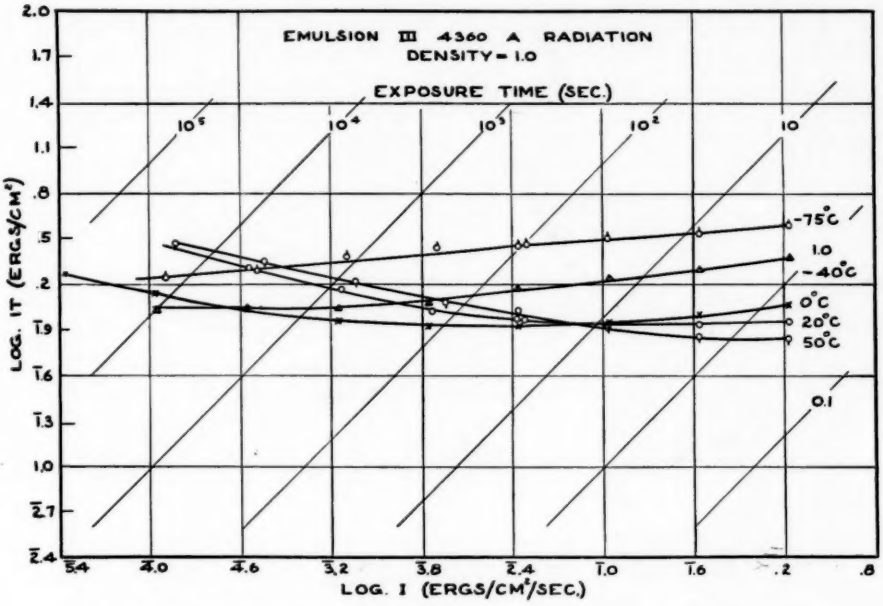

FIG. 5. Variation of the reciprocity characteristic with temperature for emulsion III.

quantitative differences, the behavior of the curves of all these emulsions is substantially the same. According to each set of curves there appear to be two definite effects which are characteristic of the behavior of the curves at reduced temperature. One of these is a vertical shift upward of the entire curve, and the other, which at first sight appears to be a skewing effect, is simply a straightening of the reciprocity curve. The combination of these two effects, with decreasing temperature, produces the net result of lowering the sensitivity at high intensities, and produces the net result of an initial increase of sensitivity followed by a decrease in sensitivity at low intensities. Certain aspects of the behavior of these curves with temperature seem to suggest that the reciprocity characteristic may consist of two separate and distinct parts, a first part which is represented by a straight-line portion sloping downward from the high-intensity end of the diagram, and a second part, which consists of the difference between the first straight-line portion and the actual reciprocity curve. In this way the exposure represented by the second part of the curve would account for the upturn of the observed reciprocity curve at low intensities. It is seen that such a division of the curve is arrived at by noting that the reciprocity curve at lowered temperature approaches a straight line with slope independent of temperature, while the upturn of the curve at low intensities disappears with decreasing temperature. The effect of lowered temperature could, on this basis, be interpreted as causing a parallel shifting of the first straight-line component vertically upward, and a simultaneous disappearance of the second part, this second part being the part of the exposure which accounts for the upturn of the reciprocity curve at low intensities. This method of analyzing the data will be considered more thoroughly in the discussion of results which is to be presented later.

Effect of wavelength upon the temperature variation

The effect of wavelength of exposing radiation upon the temperature effect studied here may be seen from the curves of Figs. 3, 6 and 7. In these diagrams are presented reciprocity curves of

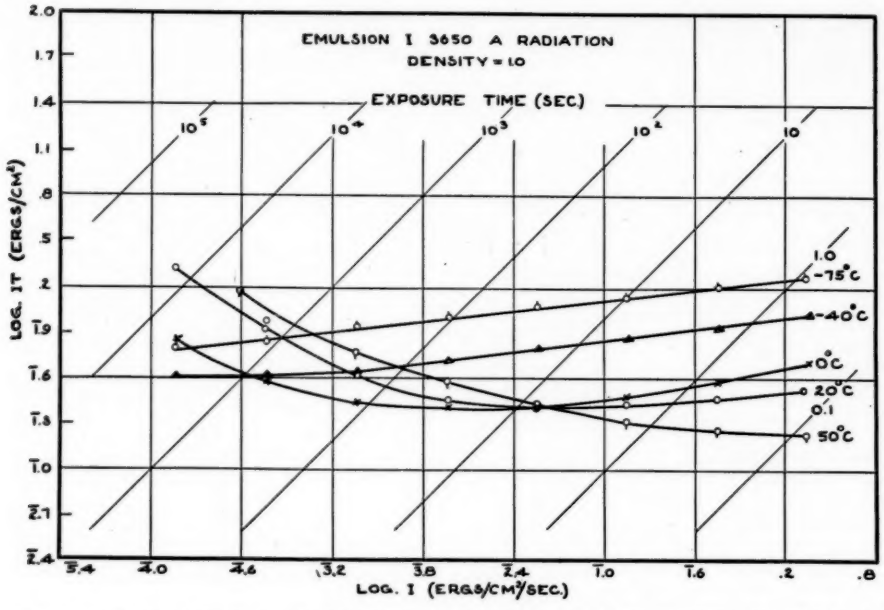

FIG. 6. Variation of the reciprocity characteristic of emulsion I with temperature, for radiation of wavelength 3650A.

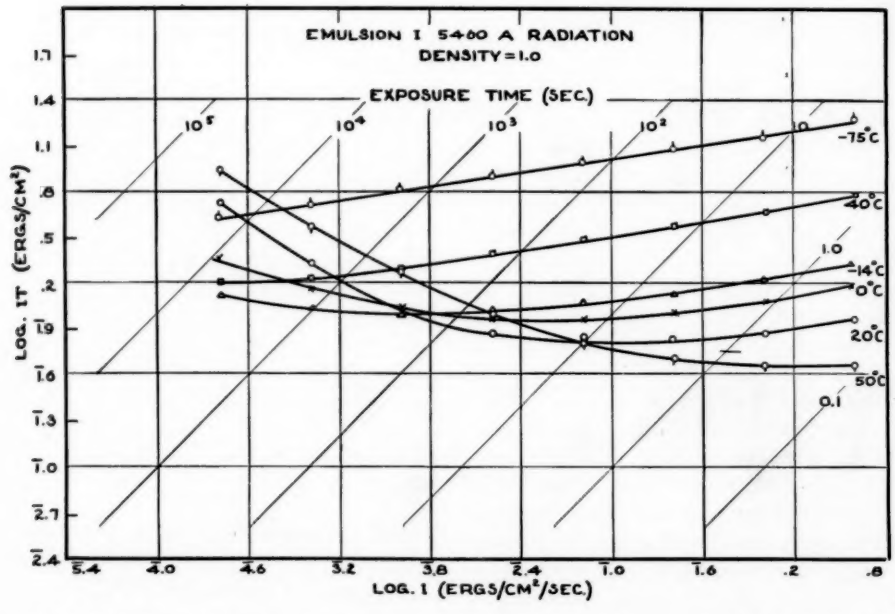

FIG. 7. Variation of the reciprocity characteristic of emulsion I with temperature, for radiation of wavelength 5460A.

emulsion I taken with radiations of wavelength 3650A, 4360A and 5460A. The variation in the reciprocity characteristic with temperature is seen to be qualitatively the same for all wavelengths. The shifting upward and the straightening of the curves with lowered temperature are features common to all three sets of curves. However there is one significant difference, this being relative to the quantitative shift upward of the reciprocity curves with lowered temperature. This shift is least for radiation of wavelength 3650A and greatest for radiation of wavelength 5460A. Evidence will be presented later which indicates that this vertical shift of the curves with temperature may be ascribed to a change in the absorption coefficient of the grain. That the change of the absorption coefficient should be greater for the less energetic radiation seems in accord with what might be expected.

Temperature-log exposure curves at constant intensity levels

Curves showing the variation of log exposure with temperature, for any level of intensity, may be obtained from the reciprocity diagram. These are obtained by plotting the log IT values (for constant density) corresponding to a single intensity level, as a function of the temperature. Curves obtained in this manner, for three levels of intensity, are presented in Figs. 8 and 9. The three intensity values for which these curves are given, correspond to the log I values: (a), 0.2; (b), $\bar{2}.4$; (c), $\bar{4}.6$. These three values include practically the entire intensity range covered by the reciprocity curves. The three sets of curves of Fig. 8 are for three different types of emulsion and were obtained with radiation of wavelength 4360A. The three sets of curves of Fig. 9 are for three different wave-lengths of radiation and were all obtained with emulsion I.

The general shape of the temperature-log exposure curve shows a uniform decrease in the log IT value required with increasing temperature at low temperatures, a minimum value at an intermediate value of temperature, and a rise with further increase of temperature. The temperature of the minimum point of the curve (maximum emulsion sensitivity) is seen to be dependent upon the intensity for which the curve is drawn. At temperatures below that of the minimum point of the curve, the curves are straight lines with slopes independent of intensity. It is to be noted that this straight-line portion of the curves occurs only in the region of temperature and intensity for which the reciprocity curve itself has become straight. The slope of the curves in this region, therefore, measures the rate at which that part of the reciprocity curve, which has become straight, is shifted vertically upward with temperature. The parallelism of the curves at different intensities is in accordance with the fact that the straight-line portion of the reciprocity curve shifts so that it remains parallel to itself. In accordance with the interpretation mentioned previously for the vertical shift of the reciprocity curve with temperature, this straight-line portion of the temperature-log exposure curve would correspond to the change in the absorption coefficient, with changing temperature. Analytically, this straight-line part of the temperature-log exposure curve may be represented by an equation of the type

$$\log IT = \log \alpha - \beta(t^\circ), \tag{1}$$

which reduces to the form

$$IT = \alpha e^{-\beta t^\circ}, \tag{2}$$

for the straight exposure value. In this equation, α and β are constants, β being the slope of the straight-line portion of the temperature-log exposure curve.

The part of the temperature-log exposure curve at and beyond the minimum point is in the region of temperature and intensity for which the reciprocity curves have not become straight lines. Therefore, the upturn of the temperature-log exposure curves is also to be attributed to the so-called second part of the exposure, which is responsible for the upturn in the reciprocity failure curves at low intensities. If this component of exposure is attributable to a definite type of reaction, it is seen that this reaction is enhanced by raised temperature as well as by lowered intensity.

An inspection of the curves of Fig. 9 shows the manner in which the slopes of the straight-line portions of the temperature-log exposure

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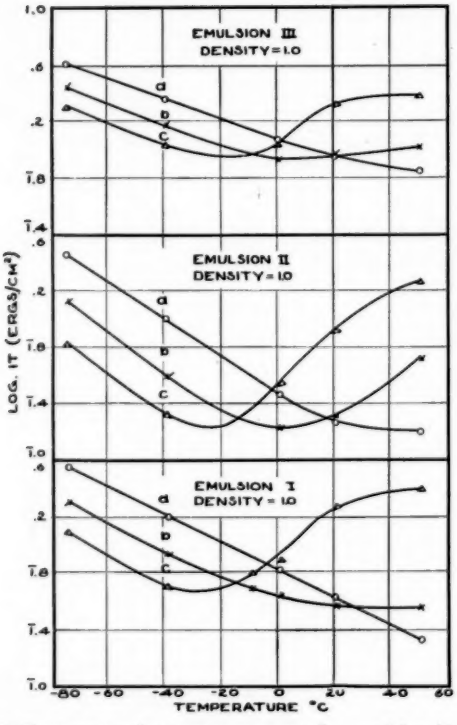

Fig. 8. Temperature-log exposure curves for emulsions I, II and III for radiation of wavelength 4360A.

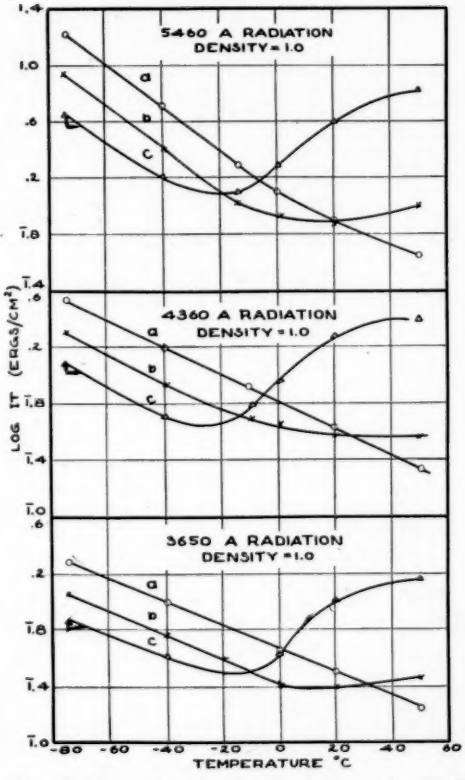

FIG. 9. Temperature-log exposure curves for emulsion I, for radiation of wavelength 3650A, 4360A and 5460A.

curves vary with wavelength of radiation. The slopes of these curves are plotted as a function of wavelength in Fig. 10. The trend of this curve indicates that the temperature coefficient is approaching, at shorter wavelengths, a constant value greater than zero.

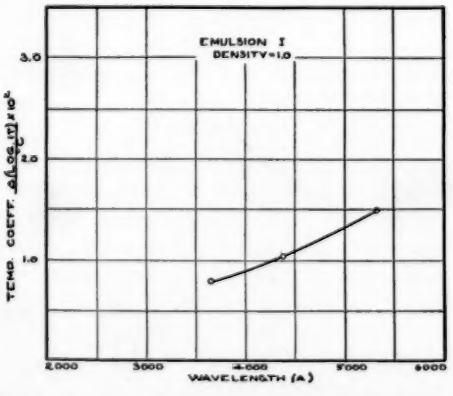

FIG. 10. Temperature coefficient of sensitivity at different wavelengths.

Effect of wavelength upon shape of the reciprocity characteristic

In a former investigation[12] in which the reciprocity failure characteristic of a single emulsion was determined for several radiations of widely different wavelengths, it was found that this characteristic did not alter in shape with wavelength. This was shown by the fact that the reciprocity curves were all parallel vertically when represented on a diagram whose abscissa was the log T, instead of log I, factor of exposure. These former experiments were made for only one temperature, 20°C. The present data, which

[12] J. H. Webb, J. Opt. Soc. Am. **23**, 316 (1933).

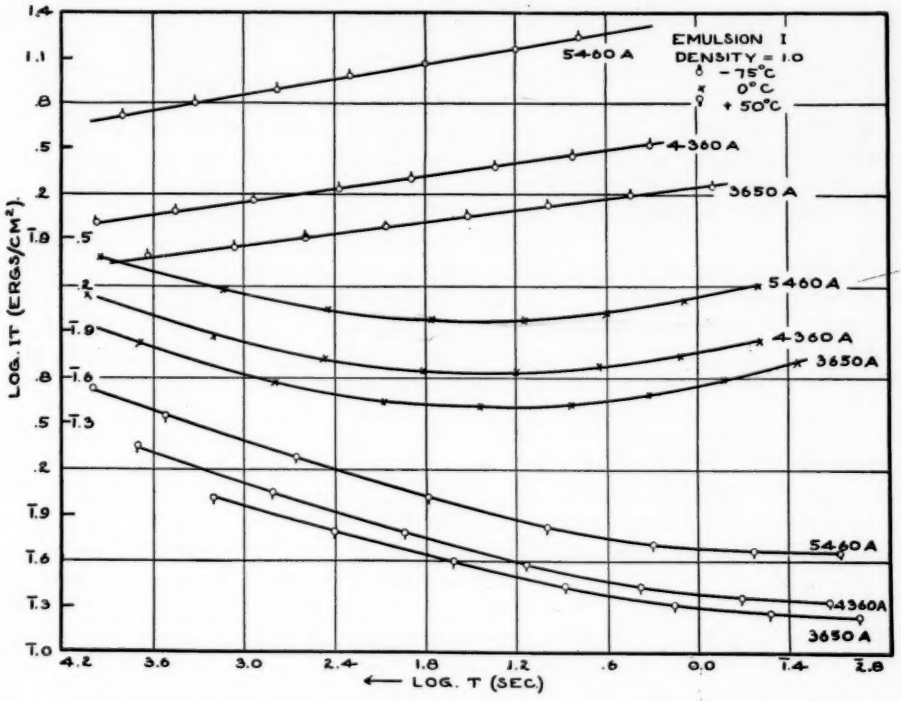

FIG. 11. Illustrating the equality in shape of the reciprocity characteristics for different wavelengths, over a wide range of temperatures.

cover a wide range of temperatures, offered a good opportunity to test this result at other temperatures. Accordingly, the reciprocity curves of emulsion I, for wavelengths 3650A, 4360A and 5460A, have been reproduced, for three different temperatures, in Fig. 11. The change in shape of these curves over the range of temperatures of −75°C to 50°C is very great, yet the curves for different wavelengths are identical in shape at each separate temperature. These findings confirm the results of the former work and extend their validity, it is believed, to all temperatures. The results are also thought to support the thesis proposed in the former paper that the mechanism of latent image formation is the same for all wavelengths. According to evidence in the earlier paper, corresponding points of the reciprocity curves for different wavelengths are points of equal rates of utilization of quanta, the vertical parallel shift upward of the reciprocity curves with wavelength indicating that the absorption coefficient is different for the different wavelengths.

DISCUSSION OF RESULTS

The experimental results presented show that the variation in emulsion sensitivity with temperature is markedly dependent upon the intensity at which measurements of the sensitivity are made. The results also show that at any level of intensity, a gain, a loss, or a zero change in sensitivity may result from a change in temperature, depending upon the range of temperature in which the measurements are made. Further, the variation in sensitivity with temperature has been shown to be altered somewhat by the wavelength of the exposing radiation and by the type of emulsion employed. Considering all these factors, it is not surprising that the results of the early experimenters on this subject were discordant. As first pointed

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out by Wallace,[7] such discordances are attributable mainly to differences in the intensity used in their exposures. Discrepancies would also have arisen due to differences in the quality of the exposing radiation and to differences in the type of emulsion. However, differences introduced due to these causes would have been small and quantitative, rather than qualitative, in nature.

The aspect of the foregoing results, which is thought to be of chief interest, is their bearing upon the nature of reciprocity law failure and latent image formation. The results have shown that the two ends of the reciprocity curve are affected in a radically different manner by a change of temperature. As pointed out earlier, this behavior has been interpreted as suggesting that the reciprocity curve may consist of two component parts, which correspond to two modes of utilization of exposure in the formation of the latent image, and that the alteration in shape of the reciprocity curve with change of temperature is to be attributed to the different ways in which these modes of utilization of exposure are affected by temperature. A more thorough analysis of the data, by means of which such a separation of the reciprocity curves into component parts has been effected, will now be given.

In this analysis, the shape of a reciprocity failure curve will be interpreted as being characteristic of an activity relative to the disposition of those quanta accepted by the grain. Accordingly, any component of the reciprocity curve which permits itself to be experimentally distinguished (as with temperature change) by virtue of the fact that its shape remains the same though it is shifted, will be interpreted as representing a component of exposure corresponding to one type of activity in the formation of the latent image. In order to make the analysis definite, the discussion will be confined to the curves of Fig. 7.

It may be seen from an inspection of these curves, that the transformation in the reciprocity characteristic which occurs in passing from 20°C to −75°C consists of a shift upward and at the same time a straightening of the curve. It is to be noted that the reciprocity characteristic has become straight for a considerable portion of its length at −40°C, and it is seen that the change experienced by this portion after becoming straight consists of a parallel shifting vertically upward. This behavior of the reciprocity curve with lowered temperature is the criterion used for separating the reciprocity curve into component parts. The component parts are selected on the basis of the fact that they represent constituent parts of this curve which are altered in different ways by changes in temperature. The approach of the reciprocity curve at low temperatures to a straight line which subsequently shifts parallel to itself is interpreted to mean that this straight-line curve represents one of the components of exposure, corresponding to one type of activity in the formation of the latent image. The disappearance of the upturn in the reciprocity curve at low intensity with lowered temperature affords the criterion for selecting the second component of the reciprocity curve. It may be seen that the two components of the reciprocity curve chosen in this manner are affected in entirely different ways by temperature change, one of the components having a positive temperature coefficient, while the other has a negative temperature coefficient.

The separation of the exposure into two component parts as outlined above has been graphically carried out for the 20°C reciprocity curve, and the method of procedure is illustrated in Fig. 12. The reciprocity curves shown in this diagram are the 20°C and the −75°C curves of Fig. 7. The graphical method of separating the curve into its component parts is as follows: First, the curve AB is drawn parallel to the −75°C curve, and tangent to the 20°C curve at the point where the 20°C curve becomes parallel to the −75°C curve. This straight line AB represents one of the components of exposure, while the curve representing the second component of exposure is obtained by subtracting the exposure represented by curve AB from that represented by the 20°C reciprocity curve. It is to be noted that this cannot be done by subtracting the ordinates on the reciprocity diagram directly, since this would give a curve representing the logarithm of a factor by which curve

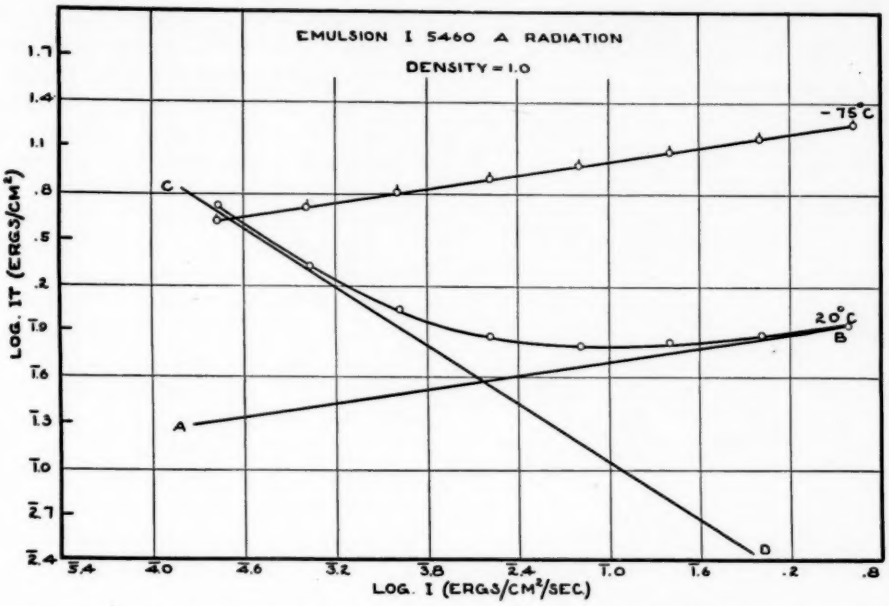

FIG. 12. Illustrating the manner of subdivision of the reciprocity characteristic into two component parts.

AB should be multiplied to give the 20°C curve. Instead, the actual exposure values must be subtracted and the logarithm of this difference of exposure plotted. This gives the curve *CD* which represents the logarithm of the actual exposure values of this second component. The fact that the curve *CD* results in a straight line may appear contradictory in view of the fact that the reciprocity curve is a curved line. However, these facts are seen to be consistent when it is recalled that the reciprocity curve is not a straight sum of the curves *AB* and *CD* but is obtained by taking logarithms of the sums of the exposures represented by the two curves, thus,

$$\log (IT)_{20°C} = \log [(IT)_{AB} + (IT)_{CD}].$$

That the curve *CD* results in a straight line for this case was at first thought to be accidental, but a similar construction for all the other reciprocity curves of Fig. 7 showed that the curve *CD* representing the second component of exposure came out in each case practically as a straight line. This straight-line feature of the curve *CD* is not considered of especial significance, however, because this curve is actually to a log scale and a straight line does not necessarily indicate a simple relationship between exposure and intensity. It will be seen later, however, that this feature is useful in the analytical representation of the reciprocity curve. It is to be noted in regard to curve *CD* that its slope as well as its height decreases with lowered temperature. The decrease in slope shows that the effect of temperature is dependent upon intensity level, while the decrease in height shows that the curve *CD* represents a component of exposure corresponding to a reaction which decreases with lowered temperature. At sufficiently low temperature this reaction entirely disappears. This, together with the fact that its temperature coefficient is of the right order of magnitude, is considered as evidence that this reaction is of a chemical nature. The temperature coefficient is portrayed by the curves of Fig. 13. These curves were plotted for a fixed intensity of exposing radiation (log $I = \bar{4}.6$), and represent the amount of exposure corresponding to the component *CD*

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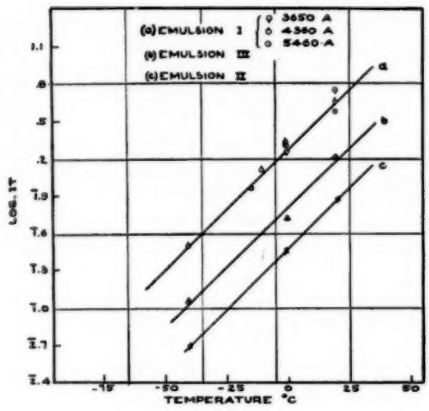

FIG. 13. Curves showing the amount of exposure used in the reaction CD as a function of temperature.

as a function of temperature. The three curves, (a), (b) and (c), pertain to the three types of emulsion used in these experiments. The three sets of points of curve (a) were obtained for different wavelengths and, in order to compare the data taken for these three different radiations, the sets of points for 4360A and 3650A were shifted in such a manner as to make their $-40°C$ points coincide with the $-40°C$ point taken with the 5460A radiation. The 50°C points were omitted from these curves as there appeared some evidence of growth of the image from heat alone in the long exposures at this temperature. The slopes of these curves show that the exposure represented by the component *CD* for this intensity doubles for each 15°C rise in temperature. The magnitude of this temperature coefficient, as well as the wide range of temperature over which it holds, is considered as supporting evidence for the view that the reaction represented by the component curve *CD* is purely chemical in nature. The fact that the curves for all three emulsions have practically the same slope suggests the possibility that the curve *CD* is representative of a common reaction for all three cases.

The present results offer no direct explanation of the types of reaction represented by the components of exposure *AB* and *CD*. The fact that the reaction *CD* can be completely eradicated by lowered temperature suggests that it is entirely chemical. Also, as this reaction practically disappears at high intensities, it is apparently not essential to the formation of the latent image. The simplest interpretation appears to be that the curve *CD* represents a component of exposure which does not contribute to the developable image. The slope of the curve *CD* indicates that this reaction is subject to a reciprocity law failure, but no clue as to the cause of this failure is given by the present results. Relative to the component of exposure represented by the curve *AB*, the effect of lowered temperature is to produce a vertical parallel shift upwards of the curve. That the curve remains parallel to itself has been taken as evidence that it represents exposure used in a single type of reaction, and the fact that the curve shifts upward with lowered temperature indicates that this reaction is probably not of a chemical nature. Certain aspects of the behavior of the component curve *AB* make it appear that the shift of this curve with temperature is due to a change in the absorption coefficient of the grain. Chief among these is the fact that the shift of the curve *AB* with temperature is exactly similar to that of the reciprocity failure curves for varying wavelength,[12, 15] in which case it has been rather definitely shown that the change occurring is due to a change in the absorption coefficient. Also, direct experimental measurements are presented later in this paper which furnish evidence that the shift of curve *AB* is to be attributed to a change in the absorption coefficient of the grain rather than to an increase in the actual number of quanta utilized. Undoubtedly, part of the exposure represented by the curve *AB* can be charged to the production of the silver which composes the developable centers of the latent image, but further than this, little can be said concerning the disposition of this exposure. As the curve *AB* is not horizontal, it too exhibits a reciprocity law failure. The positive slope of this curve indicates an increase in the number of quanta utilized as the intensity is increased. The present results offer no information concerning the cause of this failure except that it is independent of temperature.

Theoretical formulation of the reciprocity characteristic

The finding in the foregoing analysis that the reciprocity failure curves could be synthesized by taking the logarithm of the sums of the exposure values represented by the ordinates of two straight-line curves suggested the possibility of deriving a simple theoretical formulation of the reciprocity curves in terms of these two component parts. For this purpose (see Fig. 12) curve AB can be represented by the straight-line equation of the form,

$$\log (IT)_{AB} = \log A + B \log I, \tag{3}$$

and the curve CD by the equally simple equation of a straight line of negative slope,

$$\log (IT)_{CD} = \log C - D \log I, \tag{4}$$

in which A, B, C and D are constants, B and D being the slopes of the respective straight lines. Ordinates of the reciprocity failure curve will be given by the logarithmic summation,

$$\log IT = \log [(IT)_{AB} + (IT)_{CD}]. \tag{5}$$

Taking anti-logarithms of Eqs. (3) and (4) and substituting the values of $(IT)_{AB}$ and $(IT)_{CD}$ so obtained, in Eq. (5), the resultant equation is,

$$\log IT = \log (AI^B + CI^{-D}). \tag{6}$$

Eq. (6) takes on a more simple form for practical use if the optimal intensity value (intensity of minimum exposure) is introduced as one of the constants. The value of I_0 in terms of the other constants is readily found by setting the differential of Eq. (6) equal to zero and solving the resultant equation, thus,

$$\frac{d(\log IT)}{dI} = \left[\frac{BAI^{-B-1} - DCI^{-D-1}}{AI^B + CI^{-D}}\right]_{I=I_0} = 0,$$

from which

$$A/C = (D/B) I_0^{-D-B}. \tag{7}$$

If now the equation,

$$\log I_0T_0 = \log (AI_0^B + CI_0^{-D})$$

be subtracted from Eq. (6), it is found that

$$\log \frac{IT}{I_0T_0} = \log \frac{(A/C)I^B + I^{-D}}{(A/C)I_0^B + I_0^{-D}}. \tag{8}$$

Substituting in (8) the value of A/C from (7), the equation

$$\log \frac{IT}{I_0T_0} = \log \frac{B}{D+B}\left[\frac{D}{B}\left(\frac{I}{I_0}\right)^B + \left(\frac{I}{I_0}\right)^{-D}\right] \tag{9}$$

follows.

Eq. (9) is a sort of generalized equation of a catenary, and is a form which was once suggested by Kron[13] for representing the reciprocity law failure. By setting the constant B equal to D, the Eq. (9) reduces to the simple catenary equation used so extensively[14] by different investigators for representing the reciprocity law failure characteristic.

It is of interest to note that Eq. (9) permits a steep rise of the reciprocity curve in the region of low intensities, where the simple catenary equation of Kron and Halm gives a curve which is too flat and usually deviates considerably from the experimental curve.

The Eq. (9) gives a family of curves which fit all the experimental curves of Fig. 7 within experimental error when the constants in this equation are given the values shown in Table I.

TABLE I.

Temperature	B	D	Log I_0T_0	Log I_0
50°C	0.181	0.55	$\bar{1}.66$	0.2
20°C	0.181	0.658	$\bar{1}.8$	$\bar{2}.8$
− 0°C	0.181	0.457	$\bar{1}.96$	$\bar{2}.6$
−14°C	0.181	0.377	$\bar{1}.98$	3.6
−40°C	0.181	0.33*	0.2*	—
−75°C	0.181	0	—	—

* Subject to large error in determination.

In the case of the −75°C curve, the Eq. (9) must be used in a limiting form. Assuming in this case $I \gg I_0$, Eq. (9) can be written

[13] E. Kron, Publ. Astrophys. Obs. zu Potsdam, No. 67, (1913).

[14] E. Kron, reference 13; J. Halm, Monthly Notices Roy. Astron. Soc. **75**, 150 (1915); **78**, 379 (1918); L. A. Jones and E. Huse, J. Opt. Soc. Am. **7**, 1079 (1923); **11**, 319 (1925); L. A. Jones, V. C. Hall and E. Huse, ibid. **12**, 321 (1926); L. A. Jones and V. C. Hall, ibid. **13**, 443 (1926); L. A. Jones, V. C. Hall and R. M. Briggs, ibid. **14**, 223 (1927).

$$\log IT = \log \frac{D}{D+B} I_0 t_0 + B \log \frac{I}{I_0}, \qquad (10)$$

the term $(I/I_0)^{-D}$ being neglected in comparison with the term $(I/I_0)^B$. From Table I, it is to be noted that of the two curves representing the two components of exposure, the slope of curve CD alone changes with temperature.

This semi-empirical derivation of the theoretical expression (9) for representing the reciprocity characteristic will show that the present analysis of reciprocity law failure fits in very well with former experimental work on this subject in which an equation of the type of (9) has been used in a purely empirical manner.

Experimental confirmation of analysis

Up to the present point, the division of the reciprocity failure curve into two component parts, as outlined in the foregoing analysis, has been somewhat arbitrary, the mode of division having been suggested by certain qualitative features of the behavior of the reciprocity curve at lowered temperature. The fact that the method of division into the component parts possesses a certain amount of uniformity and consistency over a wide range of temperatures and is further capable of yielding an equation which represents the reciprocity curves quite accurately is thought to lend considerable weight to this mode of interpretation. There is one experiment, however, which is believed to show through quantitative evidence that this mode of division of the exposure actually corresponds to a division in the mode of utilization of the quanta by a grain. The theory and results of this experiment will now be given.

For purposes of explanation, the two reciprocity curves of Fig. 12 have been redrawn in Fig. 14 as a function of the log T, instead of the log I, factor of exposure. The curve AB is again drawn parallel to the -75°C curve, and represents one component of exposure of the 20°C curve. If there were no second component of exposure, curve AB would represent the total exposure required at 20°C to produce the given density. Point b'' would accordingly represent a point at which the rate of utilization of quanta has a definite value, and less than that for a point, such as b', by an amount determined by the rate of utilization of quanta in the second component of exposure. If, according to the analysis, the vertical shift of the curve AB is due to a change of the absorption coefficient of the grain rather than to a change in the number of utilized quanta, then point b on the -75°C and point b'' on the curve AB are to be interpreted as points of equal rates of utilization of quanta. The rate of utilization of quanta for point b should be, according to this, less than the rate of utilization of quanta for point b' by an amount equal in log units to the intercept C. Therefore, if an experimental determination of the rates of utilization of quanta for points b' and b yields the result that the rate for point b' is greater than that for b by an amount equal to that predicted, it will be considered as supporting evidence for the foregoing analysis. A determination of the rates of utilization of quanta for points b' and b is possible and the experiment by means of which this has been carried out will now be described.

The rate of utilization of quanta is measured for any point on the reciprocity curves by measuring the critical frequency of flash, f_c, of an intermittent exposure. This is believed established by former experiments[15] and the theory has been fully treated in a separate paper.[16] According to the theory, the critical frequency of flash f_c is given by the formula

$$f_c = (1/x_0)na, \qquad (11)$$

in which x_0 is a pure number, its value being dependent upon the ratio of light to dark of the intermittent exposure, a is the area in square centimeters of an average photographic grain, and n is the average number of quanta incident per square centimeter per second on the grain. In the derivation of the above formula, it was assumed that the entire area of the grain was susceptible to light. If only a fractional area of the grain, ϵ, is susceptible, then the factor a in the formula should be replaced by ϵa, thus,

$$f_c = (1/x_0)\epsilon na. \qquad (12)$$

[15] J. H. Webb, J. Opt. Soc. Am. **23**, 157 (1933); **23**, 316 (1933).

[16] L. Silberstein and J. H. Webb, Phil. Mag. **18**, 1 (1934).

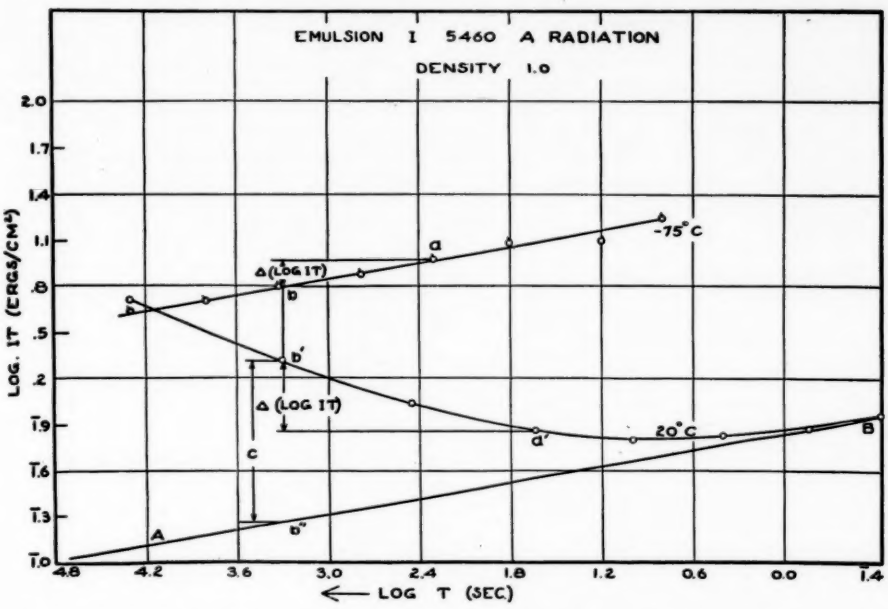

FIG. 14. Construction to illustrate the number of quanta actually utilized in the second component of exposure.

A little reflection will show that this factor ϵ can be interpreted as representing either the absorption coefficient or the fractional area of a grain susceptible to quanta. These two physical factors are numerically equal, either being a measure of the percentage of quanta utilized by a grain.

The factor $1/x_0$ in formula (12) was discussed fully in the above-mentioned paper[16] devoted to the theoretical treatment of this subject. Its value is somewhat arbitrary within narrow limits and depends to a certain extent upon the experimental accuracy claimed in this type of measurement. In the present work, however, since this formula is used chiefly to determine relative values of the quantity ϵ in two experiments for which the factors $1/x_0$ and a are kept constant, this uncertainty is of small moment. In the one case where the formula is used to determine the absolute value of ϵ, the factor $1/x_0$ is given the value of 2.5 in accordance with the criterion laid down for its determination in the theoretical treatment.[16] This value of $1/x_0$ applies to a light to dark ratio, 1 : 15, which was the value used in experiments performed here.

The critical frequency of flash is determined experimentally by measuring the frequency of flash for which the intermittency effect disappears. Experiments for the determination of this quantity were carried out for the two temperatures -75°C and 20°C, and the experimental curves from which the values of f_c were determined are shown in Fig. 15. It was possible to choose in advance from the reciprocity curves of Fig. 14 the values of intensity, n, which are necessary to give for both temperatures equal densities in equal overall times. The approximate values of the critical frequency of flash as read from the curves of Fig. 15 are 0.25 flashes/sec. for the -75°C curve, and 4 flashes/sec. for the 20°C curve. These values of f_c are interpreted as proportional to the rate at which quanta are utilized by a grain, and indeed as equal to this rate except for a division by the factor $1/x_0$ in the formula, $f_c=(1/x_0)\epsilon na$. This is seen by noting that ϵna is simply the rate at which effective quanta strike the area a. That f_c corresponds physically to the rate of utilized quanta, is believed established by the fact that its very

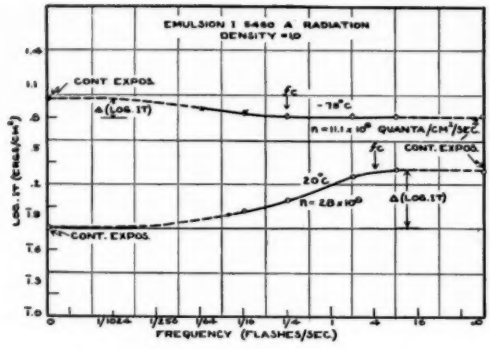

FIG. 15. Intermittency effect curves, at different temperatures, showing critical frequency of flash. (a) Curve for −75°C; (b) curve for 20°C.

determination depends upon reciprocity law failure, and reciprocity law failure must be dependent upon those quanta which actually do something.

A comparison of the two values of f_c determined in the experiments at −75°C and 20°C (for points b and b') shows the relative values of these quantities to be

$$(f_c)_{20°C}/(f_c)_{-75°C}=4/0.25=16.$$

According to prediction, the logarithm of this ratio should be equal to the logarithmic intercept value C of Fig. 14. The intercept C in this figure is seen to be 1.06, whereas the logarithm of the above ratio is equal to 1.2. The agreement obtained is considered sufficiently close to be a fairly satisfactory verification of the analysis in the light of the precision with which the value of f_c can be determined from the curves of Fig. 15.

The remaining theoretical prediction which may be checked by the experiment described above is the ratio of the values of the absorption coefficient, ϵ, at the two temperatures. If this ratio agrees with the shift in the component curve AB, it will be considered as evidence that the shift of this curve is due purely to a change in the absorption coefficient. By writing the formula (12) for two temperatures, and dividing, the equation

$$(f_c)_1/(f_c)_2=\epsilon_1 n_1/\epsilon_2 n_2 \qquad (13)$$

is obtained. Identifying the subscripts (1) and (2) with −75°C and 20°C points, and inserting the values for f_c and n from Fig. 15 in this equation, the ratio of the ϵ values for these two temperatures is found to be

$$\frac{\epsilon_2}{\epsilon_1}=\frac{n_1(f_c)_2}{n_2(f_c)_1}=\frac{11.1\times10^8}{2.8\times10^8}\,\frac{4}{0.25}=63.$$

The logarithm of this ratio is 1.8. The shift in the component curve AB in log units, for the corresponding change of temperature, is seen to be 1.5. The agreement obtained is again considered satisfactory as the value of ϵ_2/ϵ_1 depends also upon the experimental determination of f_c, which is subject to an error of as much as a factor of two.

Absolute values of ϵ_1 and ϵ_2 may be obtained from equation (12) by the insertion of numerical values for a and $1/x_0$. If this is done, the results are

$$\epsilon_1=\frac{(f_c)_1}{n_1 a(1/x_0)}=\frac{0.25}{11.1\times10^8\times0.5\times10^{-8}\times2.5}=0.018,$$

and

$$\epsilon_2=\frac{(f_c)_2}{n_2 a(1/x_0)}=\frac{4}{2.8\times10^8\times0.5\times10^{-8}\times2.5}=1.14.$$

The fact that the absorption coefficient comes out slightly greater than unity in the second case can be attributed either to an error in f_c or to the choice of too small a value for a. Accurately speaking, the value of a should not be taken as the average size of an emulsion grain, but should be taken somewhat larger than average size. However, since experiments of the above type can never yield results of very high accuracy because of the great complication of the grain size distribution, such slight discordances are not considered of importance. The very large change in the value of ϵ with temperature is interpreted as merely signifying a large change in the absorption coefficient.

SUMMARY

1. Measurements of the effect of temperature variation on the reciprocity failure curves were made on three different types of emulsion for radiation of wavelength 4360A, and on a single type of emulsion for radiations of wavelength 3650A, 4360A and 5460A. From these reciprocity curves are derived graphs showing the variation of

emulsion sensitivity with temperature at different levels of intensity.

2. A comparison of the reciprocity curves of a single emulsion taken at different wavelengths, shows these curves to be identical in shape at each temperature, although the change in shape of these curves with temperature over the range of 50°C to −75°C, is very great.

3. An analysis of the behavior of the reciprocity failure curves with varying temperature has suggested a mode of division of these curves into two component parts which, presumably, correspond to two different modes of utilization of the exposure in the formation of the latent image.

4. A theoretical formulation based on the above analysis yields the Kron-Halm catenary equation for representing the reciprocity failure curves.

5. The results of an experiment dealing with intermittent exposures are described which are considered as confirming, in some measure, that the manner of division of the exposure in the analysis, actually corresponds to a division in the mode of utilization of the exposure by the photographic grain.

In conclusion, the author desires to express his appreciation to Dr. W. Clark, Dr. K. C. D. Hickman and Mr. A. P. H. Trivelli for helpful discussion on latent image theory in connection with this work; to Dr. B. H. Carroll for a number of helpful suggestions in connection with the presentation of the results; and especially to Dr. L. A. Jones for his generous help and valuable criticism in the course of preparation of this paper.

Notice

THE Division of Industrial and Engineering Chemistry now has available a number of reprints of the Symposium on "Diffusional Processes" which was held at the Cleveland Meeting of the American Chemical Society last September.

Outstanding authorities presented papers for this Symposium. The titles and authors of the papers are as follows:

Applications of the Film Concept in the Petroleum Refining Industry by C. C. Monrad, Standard Oil Co. (Ind.)

Diffusion of Vapors Through Gas Films by T. K. Sherwood and E. R. Gilliland, Massachusetts Institute of Technology.

Effect of Concentration and Viscosity of Caustic Solutions Upon Rate of Absorption of Carbon Dioxide by Lauren B. Hitchcock, University of Virginia.

The Rate of Mixing of Gases in Closed Containers by Allen S. Smith, Amarillo Helium Plant, U. S. Department of the Interior.

Packing Material for Fractionating Columns by M. R. Fenske, C. O. Tongberg and D. Quiggle, Pennsylvania State College.

Moisture Movement by Capillarity in the Drying of Granular Materials by E. W. Comings and T. K. Sherwood, Massachusetts Institute of Technology.

A Method for the Design of Cooler-Condensers for Mixtures of Vapors and Inert Gases by A. P. Colburn and O. A. Hougen, E. I. duPont deNemours and Company.

The Prediction of Mass Transfer (Absorption) Coefficients from Data on Heat Transfer and Fluid Friction by A. P. Colburn and T. H. Chilton, E. I. duPont deNemours and Company.

The cost per copy is $1.00 ($1.50 by International Money Order outside of the United States); remittance may be sent to Mr. Erle M. Billings, Secretary of the Division of Industrial and Engineering Chemistry, 343 State Street, Rochester, New York.

A Maxwell Triangle Yielding Uniform Chromaticity Scales*

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CONTENTS

I. Introduction

II. Mathematical analysis

III. Mixture diagram

IV. Comparison with experimental data

A. Sensibility to change in dominant wavelength at constant purity
1. Sensibility at unit purity
2. Ratio of sensibility at purity, p, to that at unit purity

B. Sensibility to change in purity, dominant wavelength constant
1. Sensibility as a function of purity
2. Sensibility near zero purity
3. Number of steps between zero and unit purity as a function of wavelength

C. Sensibility to change in color temperature

D. Sensibility to change in Lovibond number
1. Sensibility to change in Lovibond yellow
2. Sensibility to change in Lovibond red combined with 35-yellow

E. Conclusions from the comparisons

V. Applications of the triangle

VI. Summary

VII. Appendix

A colorimetric coordinate system has been found by trial and error whose Maxwell triangle has the useful property that the length of any line on it is a close measure of the chromaticity difference between the stimuli represented at the extremes of the line. Such accurate chromaticity scales may be derived from this triangle merely by stepping off equal intervals on it that it has been called the "uniform-scale triangle." The definition of the system is given, and also a comparison of experimental sensibility data with corresponding data derived from the triangle. An important application of this coordinate system is its use in finding from any series of colors the one most resembling a neighboring color of the same brilliance, for example, the finding of the nearest color temperature for a neighboring non-Planckian stimulus. The method is to draw the shortest line from the point representing the non-Planckian stimulus to the Planckian locus.

I. INTRODUCTION

IN a previous paper[1] experimental data on chromaticity sensibility were related to the (r, g)-plot of the O. S. A. tristimulus coordinate system. The method of relation was called the "square construction" because by that method the locus of points representing chromatic colors equally distinguishable from any one given color of the same brilliance is a square centered on the point representing the given color.

The present method of relating chromaticity sensibility to the Maxwell triangle is the graphically more convenient "circle construction." When applied to the O. S. A. colorimetric coordinate system, the circle construction resulted in poorer agreement with sensibility data than the analytically simple square construction. A new coordinate system has been derived, however, for the sole purpose of making use of the circle construction to the best advantage on the equilateral Maxwell triangle. This paper gives the definition of the new tristimulus coordinate system, shows the degree of agreement obtained both with the data previously

* Publication Approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

[1] D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J. Opt. Soc. Am. **22**, 72 (1932).

assembled and those which have become available since, and points out the uses to which the Maxwell triangle of this coordinate system may be put.

II. Mathematical Analysis

If any two color stimuli specified by points (r_1, g_1, b_1) and (r_2, g_2, b_2) on the equilateral triangle are compared with brightnesses equalized, the circle construction states that the chromaticity difference, ΔE, between them is proportional to the distance, D, between the points representing them, or:

$$K\Delta E = D, \tag{1}$$

where K is a constant of proportionality dependent on the unit (least perceptible difference, probable error and so on) in which ΔE is expressed and on the experimental conditions such as field size and field brightness.[2]

From the geometry of the equilateral triangle the distance, D, may be expressed in terms of the differences between the trilinear coordinates, writing Δr, Δg, Δb as abbreviations for r_1-r_2, g_1-g_2, b_1-b_2, respectively, thus:

$$D=\{2[(\Delta r)^2+(\Delta g)^2+(\Delta b)^2]/3\}^{\frac{1}{2}}, \tag{2}$$

where unit D is the altitude of the triangle. This relation may also be written in a form particularly convenient for dealing with a number of distances in the same direction. Let Δv be any one of Δr, Δg, Δb, and Δq be either of the remaining two, then:

$$D=(2\Delta v/3^{\frac{1}{2}})[(\Delta q/\Delta v)^2+(\Delta q/\Delta v)+1]^{\frac{1}{2}}, \tag{2a}$$

$(\Delta q/\Delta v)$ constant being the condition for distances in the same direction on the triangle.

In a few cases where the circle construction is analytically inconvenient, the nearly equivalent hexagon construction has been substituted. According to this construction:

$$\begin{aligned} K\Delta E &= \Delta c \\ &= |\Delta r|,\ |\Delta g|,\text{ or }|\Delta b|,\text{ whichever is the largest;} \\ &= (|\Delta r|+|\Delta g|+|\Delta b|)/2, \end{aligned} \tag{1a}$$

where the absolute value signs ($|\ |$) indicate that the differences are to be taken all greater than zero. The hexagon construction differs from the circle construction by a factor varying from $2/3^{\frac{1}{2}}$ to 1 or in maximum by about 15 percent. It may be substituted for the circle construction if differences of this size are not significant.

III. Mixture Diagram

The coordinate system found by trial and error to yield the best agreement with chromaticity sensibility may conveniently be defined as a projective transformation of the 1931 I. C. I. standard coordinate system for colorimetry.[3] If $\bar{x}$, $\bar{y}$, $\bar{z}$ be the tristimulus specifications of a given color stimulus on the I. C. I. system, then $\bar{r}$, $\bar{g}$, $\bar{b}$, the tristimulus specifications on the uniform-scale system, may be found as:

$$\left.\begin{aligned} \bar{r} &= 3.1956\,\bar{x}+2.4478\,\bar{y}-0.1434\,\bar{z} \\ \bar{g} &= -2.5455\,\bar{x}+7.0492\,\bar{y}+0.9963\,\bar{z} \\ \bar{b} &= 0.0000\,\bar{x}+0.0000\,\bar{y}+1.0000\,\bar{z} \end{aligned}\right\}$$

The reverse transformation is given by:

$$\left.\begin{aligned} \bar{x} &= 0.24513\,\bar{r}-0.08512\,\bar{g}+0.11996\,\bar{b} \\ \bar{y} &= 0.08852\,\bar{r}+0.11112\,\bar{g}-0.09802\,\bar{b} \\ \bar{z} &= 0.00000\,\bar{r}+0.00000\,\bar{g}+1.00000\,\bar{b} \end{aligned}\right\} \tag{3}$$

For convenience in checking arithmetical results the coefficients of these transformation equations are given to at least three more decimal places than would be significant from the trial-and-error adjustment to experimentally known chromaticity sensibility.

It may be noted that the only way in which better agreement with sensibility data may be effected is by a transformation which alters the shape of the spectrum locus. Transformations which merely displace the locus with respect to the triangle, or reorient it, or expand or contract it uniformly do not at all change the agreement. The system defined by (3) is one of many possible systems giving this particular shape to the spectrum locus. The luminosity coefficients are $L_r=0.08852$, $L_g=0.11112$, $L_b=-0.09802$.

The uniform scale system is more closely allied to the O. S. A. coordinate system[4] by

[2] Judd, J. Opt. Soc. Am. **22**, 87 (1932).

[3] Proc. 8th Session, International Commission on Illumination, Cambridge, pp. 19–29, September, 1931. D. B Judd, *The 1931 I. C. I. Standard Observer and Coordinate System for Colorimetry*, J. Opt. Soc. Am. **23**, 359 (1933). T. Smith and J. Guild, *The C. I. E. Colorimetric Standards and Their Use*, Trans. Opt. Soc. **33**, 73 (1931–32).

[4] Judd, J. Opt. Soc. Am. **22**, 102 (1932).

TABLE I. *Trilinear coordinates on the uniform-chromaticity-scale triangle. Observer: 1931 I. C. I. Standard.*

Part I. For the spectrum

Wavelength (mμ)	r	g	b	Wavelength (mμ)	r	g	b
380	0.2681	0.2437	0.4881	550	0.3918	0.6072	0.0009
				560	.4404	.5592	.0004
410	.2648	.2461	.4891	570	.4995	.5003	.0002
				580	.5708	.4291	.0002
430	.2575	.2584	.4841	590	.6532	.3467	.0001
440	.2495	.2761	.4744				
				600	.7388	.2611	.0001
450	.2363	.3049	.4588	610	.8169	.1830	.0001
460	.2173	.3493	.4334	620	.8776	.1224	.0001
470	.1961	.4226	.3813	630	.9206	.0793	.0000
480	.1833	.5353	.2814	640	.9521	.0478	.0000
490	.1906	.6467	.1627				
				650	.9728	.0272	.0000
500	.2147	.7092	.0761				
510	.2440	.7249	.0312	700	1.0000	.0000	.0000
520	.2784	.7103	.0113				
530	.3145	.6805	.0050				
540	.3511	.6467	.0022				

Part II. For the Planckian radiator

Color temperature	r	g	b	Color temperature	r	g	b
100	1.0000	0.0000	0.0000	2,848	0.5434	0.4239	0.0327
300	.9981	.0019	.0000	3,500	.5103	.4427	.0470
600	.9237	.0763	.0000	4,800	.4696	.4598	.0706
1,000	.7906	.2086	.0008	6,500	.4409	.4672	.0920
1,500	.6809	.3140	.0051	10,000	.4142	.4699	.1159
1,900	.6242	.3640	.0118	24,000	.3898	.4685	.1417
2,360	.5783	.4001	.0216	∞	.3769	.4663	.1568

Part III. For some miscellaneous stimuli

	r	g	b
Abbot-Priest sunlight	0.4607	0.4665	0.0728
Abbot-Davis-Gibson mean noon sunlight	.4572	.4688	.0740
Sunlight outside atmosphere (Abbot)	.4402	.4713	.0885
Equal energy spectrum recombined	.4583	.4583	.0833

derivation than to the I. C. I. system. The coefficients of transformation between the uniform-scale system and the O. S. A. system are:

O. S. A. to Uniform-Scale	*Uniform-Scale to O. S. A.*	
1, 0, 0.1	1, 0, −0.5	
0, 1, 0.1	0, 1, −0.5	(3a)
0, 0, 0.2	0, 0, 5.0	

These coefficients have been used in transforming data on the O. S. A. basis referring to Lovibond glasses, but in the other comparisons presented, the 1931 I. C. I. standard observer and Eq. (3) have been used.

Fig. 1 shows the spectrum locus and the Planckian locus on the equilateral Maxwell triangle of the new uniform-scale system. A given color stimulus $(\bar{r}, \bar{g}, \bar{b})$ is represented on this triangle according to its trilinear coordinates (r, g, b) which are fractional parts of the total, $\bar{r}+\bar{g}+\bar{b}$, and refer to distances perpendicular to a side of the triangle. Table I gives the trilinear coordinates from which the spectrum locus and the Planckian locus on Fig. 1 have been plotted; it also gives the trilinear coordinates of the recombined equal-energy spectrum which is the basic stimulus of the I. C. I. system, Abbot-Priest sunlight,[5] mean noon sunlight from Abbot's data as averaged by Davis and Gibson,[6] and sunlight outside the atmosphere from Abbot's data.[7]

On this triangle equal chromaticity intervals are represented by lines of nearly equal length. An outstanding difference between this and the usual Maxwell triangle is the smaller distance between the Planckian locus and the spectrum locus from 550 to 600 mμ.

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[5] I. G. Priest, *A Precision Method for Producing Artificial Daylight*, Phys. Rev. **11**, 502 (1918); J. Opt. Soc. Am. and Rev. Sci. Inst. **12**, 479 (1926). D. B. Judd, *Reduction of Data on Mixture of Color Stimuli*, Bur. Standards J. Research **4**, 525 (1930); RP163.

[6] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature*, Misc. Pub. Bur. Stand. **114**, 16 (1931).

[7] Reference 6, Table I.

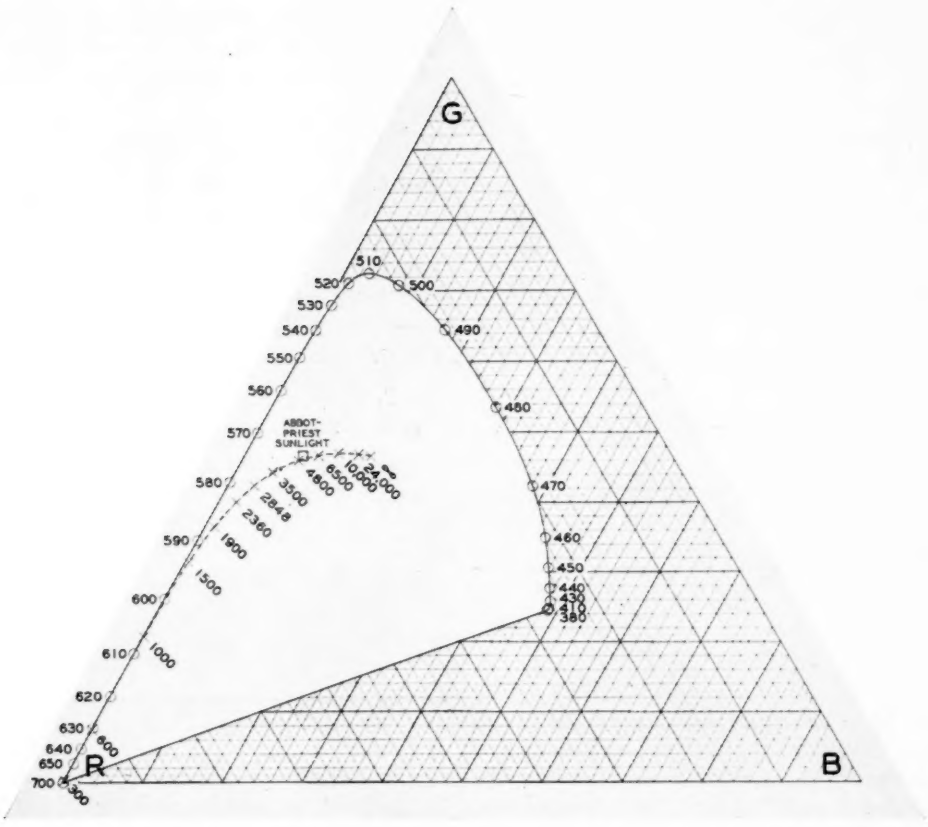

FIG. 1. The uniform-chromaticity-scale triangle. Wavelength along the spectrum locus is indicated in millimicrons; color temperature along the Planckian locus is given in degrees Kelvin.

IV. COMPARISON WITH EXPERIMENTAL DATA

As in the previous paper the method of comparison is to plot the various types of chromaticity sensibility data on the same graph with similar data derived from the triangle by relations (1) or (1a). The constant, K, is adjusted to a value providing nearly the best agreement in order to facilitate comparison.

In the derivation of the new coordinate system, considerably more weight was given to some chromaticity-sensibility data than to others. These weights were determined in no very systematic or quantitative way; they depended on a critical survey of the whole body of data and on estimates of relative reliability of the various sets of data. Of the rather large proportion of the data not taken into account in the actual derivation of the system, much was found to be in good agreement. All the data yielding sufficiently good agreement to warrant graphical comparison are shown, however, without regard to the weight given them in deriving the system. Data which do not agree well are not shown but are mentioned at appropriate places in the text.

A. Sensibility to change in dominant wavelength at constant purity

1. Sensibility at unit purity

By Eq. (1a) the hexagon construction yields for this case:

$$d\lambda/dE = K/(dc_h/d\lambda)_{p=1}, \tag{4}$$

where λ is wavelength and $(dc_h/d\lambda)_{p=1}$ is the

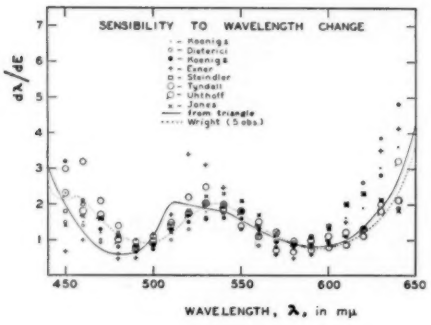

FIG. 2. Chromaticity sensibility to wavelength change in the spectrum.

largest of $|dr/d\lambda|$, $|dg/d\lambda|$, $|db/d\lambda|$ referring to the spectrum locus. Fig. 2 compares experimental values of $d\lambda/dE$ with $0.007/(dc_h/d\lambda)_{p=1}$ which is shown by the solid line. The experimental data indicated by points are those given in Fig. 3 of the previous paper;[8] to these has been added a dotted curve representing the result of a recent careful investigation with modern apparatus by Wright and Pitt.[9]

The degree of agreement shown by Fig. 2 is good but not perfect. The solid curve rises somewhat too high near 510 mμ and stays too low near 470. The curves of results by Laurens and Hamilton[10] showing pronounced secondary minima not duplicated in the solid curve have not been given; a discussion of the reliability of these omitted results is given by Wright and Pitt.[11]

2. Ratio of sensibility at purity, p, to that at unit purity

Analogous to relation (9a) of the previous paper extended to the general choice of heterogeneous stimulus[12] the hexagon construction yields:

$$\frac{(d\Lambda/dE)_p}{(d\Lambda/dE)_{p=1}}=\frac{[s_h(1-p)+s_wp]^2(dc_h/d\lambda)}{s_wp\{[s_h(1-p)+s_wp](dc_h/d\lambda)+(1-p)(c_w-c_h)(ds_h/d\lambda)\}}, \tag{5}$$

where Λ is dominant wavelength, s_h is an abbreviation for $rL_r+gL_g+bL_b$ for the spectrum at wavelength, Λ, s_w is an abbreviation for the similar sum referring to the heterogeneous stimulus ("white light"), and c as in relation (1a) is either r, g or b. As discussed in the previous paper care must be taken in applying this formula because of the multiple definition of c_h. In the numerator, c_h is taken as r, g or b according as $dr/d\lambda$, $dg/d\lambda$ or $db/d\lambda$ has the greatest absolute value for the spectrum ($p=1$). In the denominator, c_h is chosen as r, g or b according to which choice makes the absolute value of the denominator the greatest. For most values of wavelength, c_h is the same in the numerator and denominator, but not at all wavelengths.

In Fig. 3 are shown the experimental values of this ratio obtained by Tyndall[13] (circles) and those obtained by Watson[14] (crosses). The data by Watson shown on plots marked 530, 585 and 630, refer to wavelengths 527, 589 and 632 mμ, respectively. The curves shown were obtained from Eq. (5) with Abbot-Priest sunlight as the heterogeneous stimulus, that is, for $r_w=0.461$, $g_w=0.466$, $b_w=0.073$. This choice was made because the heterogeneous stimulus used by Tyndall was a color match for Abbot-Priest sunlight; that used by Watson was similar but not definitely known.

It will be noted from Fig. 3 that the agreement at three dominant wavelengths (455, 490 and 630 mμ) is nearly perfect. In the other three cases the experimentally found relation is similar to that yielded by the triangle but not identical.

[8] D. B. Judd, J. Opt. Soc. Am. **22**, 89 (1932).

[9] W. D. Wright and F. H. G. Pitt, *Hue-Discrimination in Normal Colour-Vision*, Proc. Phys. Soc. **46**, 459 (1934).

[10] H. Laurens and W. F. Hamilton, *The Sensibility of the Eye to Differences in Wave Length*, Am. J. Physiol. **65**, 547 (1923).

[11] Wright and Pitt, Proc. Phys. Soc. **46**, 466 (1934).

[12] D. B. Judd, *A General Formula for the Computation of Colorimetric Purity*, J. Opt. Soc. Am. **21**, 729 (1931); see Eq. (14).

[13] E. P. T. Tyndall, *Chromaticity Sensibility to Wave Length Difference as a Function of Purity*, J. Opt. Soc. Am. **23**, 15 (1933).

[14] W. Watson, *Note on the Sensibility of the Eye to Variations of Wave Length*, Proc. Roy. Soc. **B84**, 118 (1911).

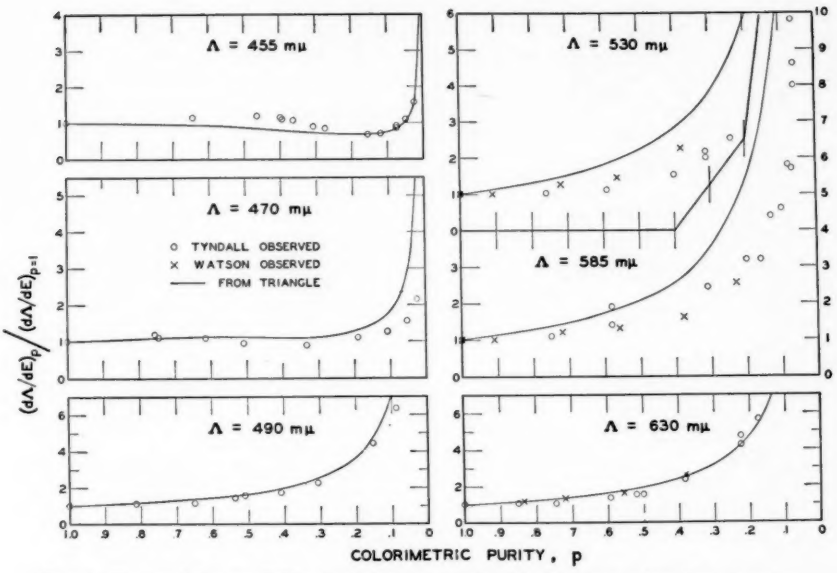

FIG. 3. Chromaticity sensibility to dominant-wavelength change as a function of purity.

B. Sensibility to change in purity, dominant wavelength constant

1. Sensibility as a function of purity

Analogous to relation (10a) of the previous paper extended as was Eq. (5) to the general choice of heterogeneous stimulus it is seen from relations (1) and (2a) that the circle construction yields:

$$\left(\frac{dp}{dE}\right)_{\Delta \text{ constant}} = \frac{K}{2}\left[\frac{3}{(\Delta q/\Delta v)^2+(\Delta q/\Delta v)+1}\right]^{\frac{1}{2}} \times \frac{[s_h(1-p)+s_w p]^2}{s_h s_w (v_h - v_w)}, \quad (6)$$

where, as in relation (2a), v is any one of r, g, b, and q is either of the remaining two; Δv may be evaluated conveniently here as $v_h - v_w$, v_h referring to the spectrum at wavelength, Λ, and v_w to the heterogeneous stimulus; Δq similarly may be found as $q_h - q_w$.

In Fig. 4 are shown a number of experimental determinations of this quantity by Donath[15] (circles) and a determination by Judd[16] (crosses). In Fig. 5 are shown all of the published experimental data on this case in the recent paper by Martin, Warburton and Morgan.[17] The curves in Figs. 4 and 5 refer to wavelengths approximately but not identically those of the experimental results and were obtained from Eq. (6), those in Fig. 4 with K adjusted in each case, those in Fig. 5 with K adjusted to 0.015 throughout. Values of K referring to Donath's work which was done with an improved technic involving rotating disks viewed by both eyes vary from 0.0014 to 0.0023; the value of K for Judd's result referring to a 2° field viewed with one eye is 0.011. The marked dependence of K on observing conditions was discussed in the previous paper.

From Fig. 4 it may be seen that with one exception (550 mμ) the experimental determination of the function agrees with that yielded by the triangle. The data shown in Fig. 5, being results of single settings rather than averages of

[15] F. Donath, *Die funktionale Abhängigkeit zwischen Reiz und Empfindung bei der Farbensättigung*, Neue Psych. Stud. **2**, 143 (1926).

[16] J. Opt. Soc. Am. **22**, 94 (1932).

[17] L. C. Martin, F. L. Warburton and W. J. Morgan, *Determination of the Sensitiveness of the Eye to Differences in the Saturation of Colours*, Medical Research Council, Reports of the Committee upon the Physiology of Vision, XIII, Special Report Series, No. 188, London, 1933.

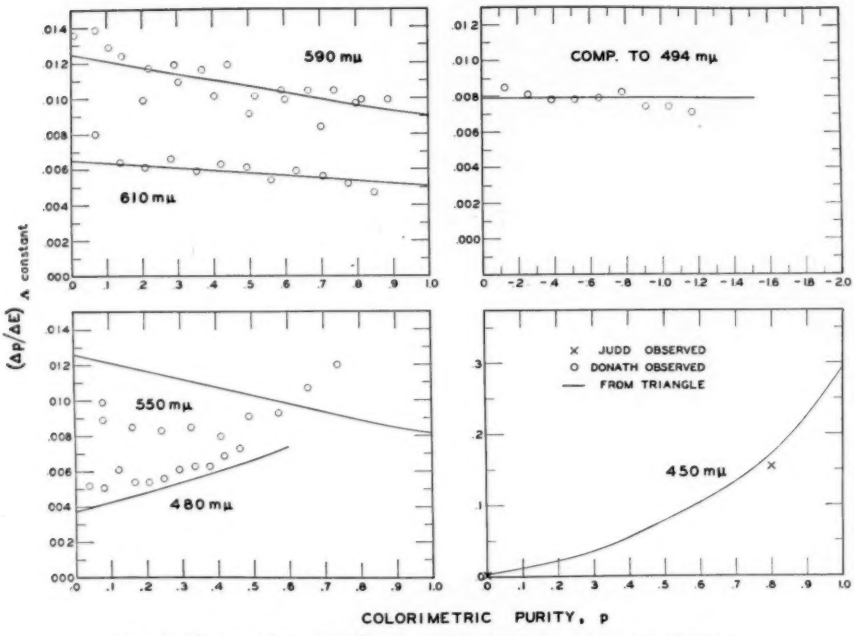

FIG. 4. Chromaticity sensibility to purity change as a function of purity.

from 4 to 10 settings, yield points that are more scattered than those of the previous figures. In general there is fair agreement between the observed function and that obtained from the triangle. Most of the points for 650 and 680 mμ are higher than the curve; perhaps, some of the discrepancy at these wavelengths is ascribable to a low field brightness for this extreme spectral region. There is a general tendency throughout for the experimental data to indicate a rise in dp/dE with purity for low purities. Some of this rise has been ascribed by the authors to a fatigue effect combined with the fact that the majority of the step-by-step settings progressed upward from zero purity; the settings in which the progression was in the opposite direction substantiate this view. It is probably significant that of all wavelengths investigated, that (460 mμ) yielding the most consistent rise for low purities is the only one for which the triangle also indicates a rise. Note also that the observed function for 546 mμ in Fig. 5 is convex upward while that in Fig. 4 for 550 mμ is convex downward, the average being in fair agreement with the function computed from the triangle.

Data by Nutting and Jones[18] and Jones and Lowry[19] deviate considerably from each other and from the functions yielded by the triangle; they are not shown graphically. Possible sources of error in these sets of data have already been discussed.[20]

Data by Judd[21] obtained through a different means (Lovibond yellow glasses) than other sets cited here deviate from the function indicated by the triangle by a factor of about 2.5. Reexamination of these data have shown that an appreciable part of the deviation can be ascribed to the two-millimicron dominant-wavelength variation from one end of the Lovibond-yellow locus to the other which was previously considered negligible. These data, therefore, give a saturation scale along the Lovibond-yellow locus, but not very exactly the scale for dominant

[18] P. G. Nutting, *The Retinal Sensibilities Related to Illuminating Engineering*, Trans. Illum. Eng. Soc. **11**, 16 (1916).

[19] L. A. Jones and E. M. Lowry, *Retinal Sensibility to Saturation Differences*, J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 25 (1926).

[20] Judd, J. Opt. Soc. Am. **22**, 95 (1932); see also footnote 17.

[21] D. B. Judd, *Saturation Scale for Yellow Colors*, J. Opt. Soc. Am. **23**, 35 (1933).

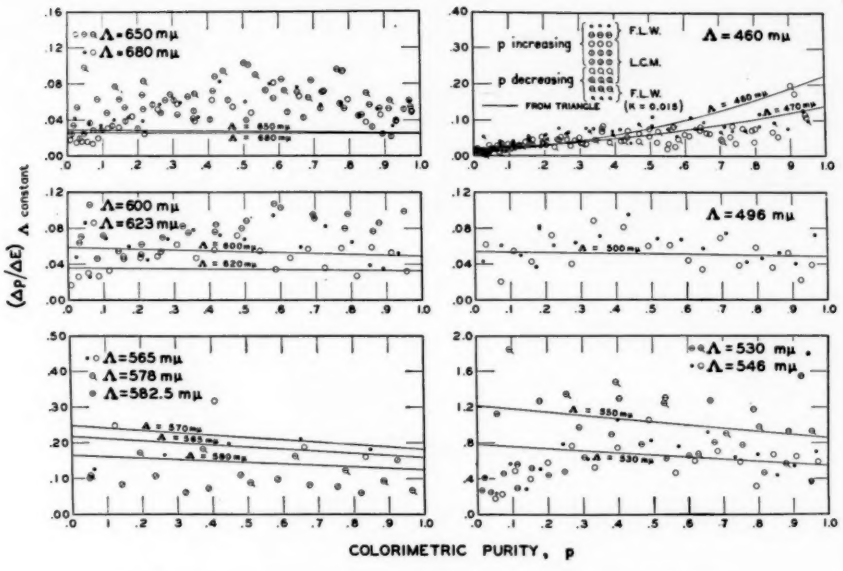

FIG. 5. Chromaticity sensibility to purity change as a function of purity.

wavelength constant at 575 mμ; hence they are not compared with the triangle in the form of dp/dE as a function of p, but as dN''/dE as a function of Lovibond-yellow number, N'' (see later section).

2. Sensibility near zero purity

From Eq. (6) it is seen that the circle construction yields:

$$\left(\frac{dp}{dE}\right)_{p\to 0} = \frac{K}{2}\left(\frac{3}{(\Delta q/\Delta v)^2+(\Delta q/\Delta v)+1}\right)^{\frac{1}{2}} \times\left[\frac{s_h}{s_w(v_h-v_w)}\right]. \quad (6a)$$

In Fig. 6 are shown experimental determinations of this quantity for a number of dominant wavelengths by Priest and Brickwedde[22] and by Purdy;[23] results by Martin, Warburton and Morgan[17] agree well with these and are not shown. These experimental values have been corrected to refer to the visibility function of the 1931 I. C. I. standard observer on the assumption that the true visibility of the three observers is the experimental mean found by Gibson and Tyndall.[24] This assumption is known to be closely correct for Priest, who was one of the 52 observers studied by Gibson and Tyndall; and it is probably nearly correct for the other

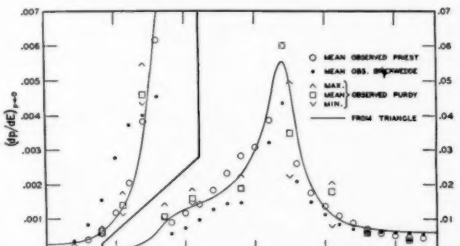

FIG. 6. Chromaticity sensibility to purity change near zero purity as a function of dominant wavelength.

[22] I. G. Priest and F. G. Brickwedde, *The Minimum Perceptible Colorimetric Purity as a Function of Dominant Wave Length with Sunlight as Neutral Standard*, J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 306 (1926); see also footnote 1.

[23] D. McL. Purdy, *On the Saturations and Chromatic Thresholds of the Spectral Colours*, Brit. J. Psych. (Gen. Sec.) **21** (Part 3), 283 (1931).

[24] K. S. Gibson and E. P. T. Tyndall, *Visibility of Radiant Energy*, Bur. Stand. Sci. Pap. **19**, 131 (1923–24); S475. See first column of their Table III, p. 174.

two (Brickwedde, Purdy). The correction was made by multiplying the reported purities by the ratio of standard I. C. I. visibility to experimental visibility (V_{ICI}/V_{exp}). This gives the least perceptible colorimetric purity of the field set by the observer as if analyzed photometrically by the standard observer.

In addition to this correction Purdy's values were multiplied by 0.20 to facilitate comparison with the Priest-Brickwedde values. The larger values found by Purdy are attributable to the fact that the size of field used by him (1.5° circular) was less than that used by Priest and Brickwedde (4° square).

The solid curve represents the right-hand member of Eq. (6a) with Abbot-Priest sunlight taken as the heterogeneous stimulus, that is, for $r_w=0.461$, $g_w=0.466$, $b_w=0.073$. This choice was made because Priest and Brickwedde used this heterogeneous stimulus, and because Purdy used one nearly like this. It will be noted that the agreement shown between the corrected experimental results and those from the triangle is good. The uncorrected purities are higher than the plotted values between 400 and 440 mμ by a factor varying from 3 to 10. The good agreement obtained with the purities corrected on the basis of the Gibson-Tyndall experimental visibility suggests that those values may be more representative than the standard values.

3. Number of steps between zero and unit purity as a function of wavelength

According to the circle construction the number of just noticeable chromaticity steps between zero and unit purity is proportional to the distance on the triangle between the point representing the heterogeneous stimulus and the spectrum locus. From relations (1) and (2) this number may be written:

$$\Delta E=(1/K)\{2[(\Delta r)^2+(\Delta g)^2+(\Delta b)^2]/3\}^{\frac{1}{2}}, \quad (7)$$

where $\Delta r=r_h-r_w$, $\Delta g=g_h-g_w$, $\Delta b=b_h-b_w$.

Fig. 7 shows this number determined experimentally by Warburton and Martin[17] for a number of dominant wavelengths with the heterogeneous stimulus a close color match for a Planckian radiator at 4800°K.

The curve on Fig. 7 gives the right-hand member of Eq. (7) as a function of dominant

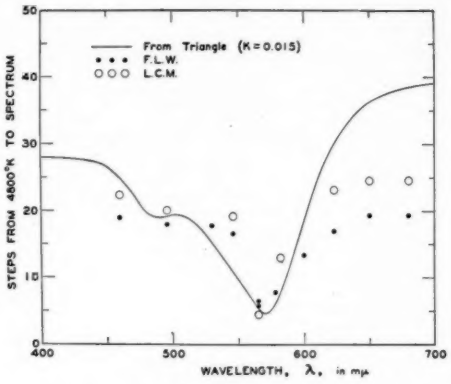

FIG. 7. Number of chromaticity steps from a Planckian radiator at 4800°K to the spectrum as a function of dominant wavelength.

wavelength for the heterogeneous stimulus used by Warburton and Martin ($r_w=0.470$, $g_w=0.460$, $b_w=0.070$). As in Fig. 5, the constant, K, is taken as 0.015. The agreement is only fair, but it may be worth noting that Martin and Warburton found more steps than indicated by the curve for the brighter part of the spectrum and fewer steps for the extremes of the spectrum. Perhaps, more steps would have been found at the extremes if examined at the same field brightness as the middle portion.

C. Sensibility to change in color temperature

An empirical formula for sensibility to change in color temperature, θ, has already been found and checked[25] over a considerable range (1800 to 11,000°K) of color temperature:

$$d(1/\theta)/dE=\text{a constant}. \quad (8)$$

This relation has been partially responsible for the proposal that reciprocal temperature be used instead of temperature for color specification of illuminants.[26]

From Eq. (1a) it is seen that the hexagon construction yields:

$$d(1/\theta)/dE=Kd(1/\theta)/dc. \quad (9)$$

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[25] D. B. Judd, *Sensibility to Color-Temperature Change as a Function of Temperature*, J. Opt. Soc. Am. **23**, 7 (1933).

[26] I. G. Priest, *A Proposed Scale for Use in Specifying the Chromaticity of Incandescent Illuminants and Various Phases of Daylight*, J. Opt. Soc. Am. **23**, 41 (1933).

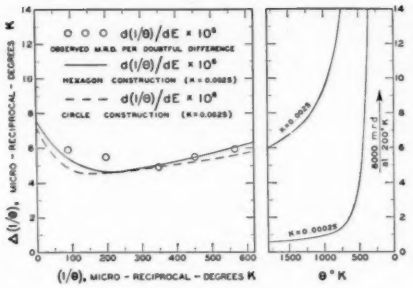

FIG. 8. Sensibility to color-temperature change.

Fig. 8 shows (solid curve) the right-hand member of Eq. (9) as a function of temperature for low temperatures (0 to 1800°K) and as a function of reciprocal temperature for high temperatures (1600°K up, or from zero to 625 micro-reciprocal degrees). Although this curve is far from being horizontal over the entire range, it is nearly horizontal over the range (1800 to 11,000°K or from about 560 to 90 m.r.d.) to which Eq. (8) applies.

Fig. 8 also shows (circles) the data by means of which Eq. (8) was verified. It is seen that the hexagon construction is also closely verified by these data. The agreement is close enough to raise the question whether the approximate hexagon construction should not be replaced by the circle construction which, from Eqs. (1) and (2), yields:

$$\frac{d(1/\theta)}{dE}=\frac{6^{\frac{1}{2}}Kd(1/\theta)}{2[(dr)^2+(dg)^2+(db)^2]^{\frac{1}{2}}}, \qquad (10)$$

where dr, dg and db refer to small differences along the Planckian locus. The right-hand member of Eq. (10) is represented on Fig. 8 by the dotted curve. It is seen that about the same degree of agreement exists with the experimental data.

D. Sensibility to change in Lovibond number

1. Sensibility to change in Lovibond yellow

Analogous to Eq. (10) the circle construction yields:

$$\frac{dN''}{dE}=\frac{6^{\frac{1}{2}}KdN''}{2[(dr)^2+(dg)^2+(db)^2]^{\frac{1}{2}}}, \qquad (10a)$$

where N'' refers to Lovibond-yellow number on the Priest-Gibson scale,[27] and dr, dg, db, refer to differences along the Lovibond-yellow locus.

In Fig. 9 are shown experimental determinations of this quantity[28] previously reported as dp/dE on the somewhat inaccurate assumption that the dominant wavelength for the Lovibond yellow glasses is constant. The curve represents the right-hand member of Eq. (10a) with K taken as 0.0045. It is seen that the agreement is fair.

2. Sensibility to change in Lovibond red combined with 35-yellow

Eq. (10a) applies to this case also if N'' is taken to refer to Lovibond-red number on the Priest-Gibson scale[29] combined with 35-yellow, and if dr, dg, db, refer to differences along the Lovibond $35Y+N''R$ locus. In Fig. 10 are shown some rather inaccurate determinations of this quantity by way of the probable error of a single setting for chromaticity match in terms of N''. These data were presented and discussed in the previous paper.[30] The curve represents the right-hand member of Eq. (10a) with K taken as 0.00013. The agreement is seen to be about as good as can be expected from data of this low precision (note point at 0.075 for $N''=17.7$). Possibly the circles indicate a curve with a somewhat greater slope than that of the curve derived from the triangle. The decrease in field brightness that accompanies an increase in N'' would account for a small part of such a discrepancy.

E. Conclusions from the comparisons

In every case the agreement previously obtained has been equalled or bettered, and in one case it has been notably improved (see Fig. 6). Agreement with data appearing since the previous paper has also been satisfactory. Considering the rather large individual-observer variation it is estimated that the uniform-chromaticity-scale

[27] I. G. Priest and K. S. Gibson, *Standardizing the Red and Yellow Lovibond Glasses*, J. Opt. Soc. Am. and Rev. Sci. Inst. **16**, 116 (1928).

[28] Judd, J. Opt. Soc. Am. **23**, 35 (1933).

[29] K. S. Gibson and G. W. Haupt, *Standardization of Lovibond Red Glasses in Combination with Lovibond 35 Yellow*, Bur. Standards J. Research, **13**, 433 (1934), RP718.

[30] Judd, J. Opt. Soc. Am. **22**, 98 (1932).

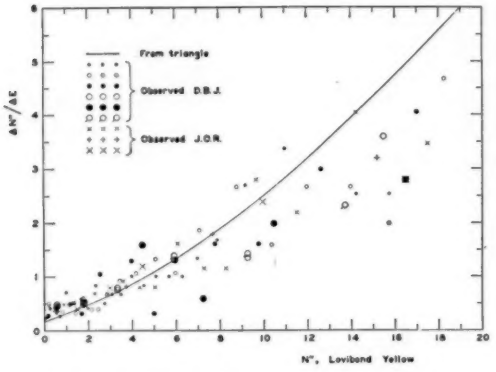

FIG. 9. Sensibility to change in Lovibond yellow.

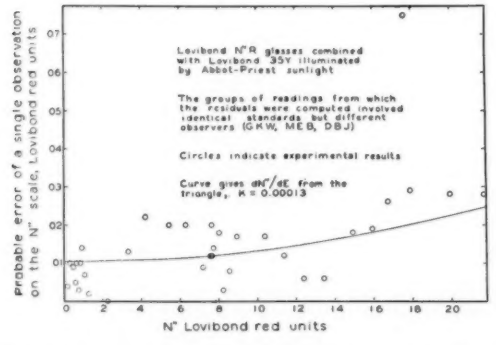
Lovibond N°R glasses combined with Lovibond 35Y illuminated by Abbot-Priest sunlight
The groups of readings from which the residuals were computed involved identical standards but different observers (G.H.W., M.E.B., D.B.J.)
Circles indicate experimental results
Curve gives dN"/dE from the triangle, K = 0.00013
Probable error of a single observation on the N" scale, Lovibond red units
N° Lovibond red units

FIG. 10. Sensibility to change in Lovibond red combined with Lovibond 35-yellow.

triangle represents about nine-tenths of available data on chromaticity sensibility within their experimental uncertainty.

V. APPLICATIONS OF THE TRIANGLE

The data presented indicate that the system chosen yields, as intended, a Maxwell triangle on which to a good approximation the length of a line is proportional to the chromaticity difference between the stimuli represented at its extremes. Although the data examined nearly all referred to small differences, it is probable that the approximation is of the same order for large differences.[31] The degree of approximation, while by no means sufficient to disprove Schrödinger's[32] theoretical conclusion that such a representation should be made on a spherical surface, is close enough to be of some theoretical interest. Since the coordinate system chosen is only one of an infinity of systems each having the same essential property, it appears possible to fit with certain restrictions the uniform-chromaticity-scale triangle to many of the theories of vision, such as the Hering opponent-colors theory, the Young-Helmholtz three-components formulation, Hecht's[33] modification of it, or the level theory of G. E. Müller.[34] In this way the implications of the uniform-scale triangle might find expression in a variety of theoretical terms. Possibly, too, the theoretical formulation which is found to accomodate the uniform-scale triangle most conveniently may be thought to have some advantage over the others.

Two practical applications of the triangle may be pointed out. One obvious use is the estimation of the chromaticity difference between any two stimuli from their tristimulus specifications. To accomplish this with maximum accuracy the tristimulus specifications must be transformed by appropriate equations such as (3) or (3a). However, the mixture diagram of the standard 1931 I. C. I. coordinate system might be made use of directly to find approximate values of chromaticity difference provided some means were available to take into account the distortion between the two diagrams. A convenient way to do this is to cover the uniform-scale triangle with a number (say 30) of equal circles; then transform by Eq. (3) these circles to the 1931 I. C. I. mixture diagram where they will appear as ellipses. The expansion or contraction of the various parts of the diagram relative to corresponding parts of the uniform-scale diagram would be indicated by the sizes, eccentricities and orientations of these ellipses.

Perhaps, the most important practical application of the uniform-scale triangle is its use in finding from any series of colors the one most

[31] J. Opt. Soc. Am. **23**, 35 (1933).

[32] E. Schrödinger, *Grundlinien einer Theorie der Farbenmetrik im Tagessehen*, Ann. d. Physik (4), **63**, 481 (1920); Die Gesichtsempfindungen, Müller-Pouillets Lehrbuch der Physik, 2nd Ed. **2**, 456 (1926).

[33] Selig Hecht, *The Interrelation of Various Aspects of Color Vision*, J. Opt. Soc. Am. **21**, 615 (1931).

[34] G. E. Müller, *Über die Farbenempfindungen*, Leipzig: Barth, 1930.

resembling any neighboring color of the same brilliance. The method, of course, is to draw the shortest line from the locus of the series to the point representing the neighboring color. This is the geometric equivalent of the method of observation on which many plans of color grading are based. The characteristic of these plans is that they pay attention to chromaticity differences along a series of color standards, and in the usual case that no member of the series gives a perfect chromaticity match, the nearest match is set and the residual difference neglected. In this way a problem which, strictly, is two-dimensional finds a one-dimensional solution that is close enough to be of practical use.

Examples of this plan are the color-grading of illuminants according to color temperature,[35] the grading of cotton-seed oil according to Lovibond red number combined with Lovibond 35-yellow,[36] and the grading of lubricating oil according to the standards of the well-known Union colorimeter.[37] Many uses of a color comparator (such as the Duboscq) having for comparison a column of liquid of variable height are more or less allied to this plan of color-grading; those involving a column producing a relatively large brightness change are little allied, but those involving chiefly changes in chromaticity are good examples of this plan.

In finding nearest color from a series the uniform-scale triangle may be used directly. The locus of the series is plotted together with the point representing the neighboring chromaticity, and the normal to the locus through the point gives the nearest chromaticity of the series. This necessitates transforming the tristimulus specification of the given chromaticity to the uniform-scale triangle each time. It is also possible to carry out the operation once for all by drawing on the uniform-scale triangle a large number of normals to the locus of the series completely covering at sufficiently small intervals the areas of the triangle near the locus. These normals could then be transformed by Eq. (3) to the standard 1931 I. C. I. coordinate system and they would permit nearest chromaticity to be found from trilinear coordinates in the standard system without the necessity for transformation each time to the uniform-scale triangle. It is planned to do this for the Planckian locus to facilitate finding nearest color temperature. Preliminary comparison of nearest color temperatures found by the uniform-scale triangle indicates good agreement with those found by Davis' empirical method.[38]

[35] I. G. Priest, *The Complete Scale of Color Temperature . . .*, Phys. Rev. **20**, 93 (1922); *The Colorimetry and Photometry of Daylight and Incandescent Illuminants . . .*, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 1175 (1923).

[36] Report of Color Committee of the American Oil Chemists' Society, Year 1932–1933, Oil and Soap **10**, 114 (1933).

[37] In this instrument no provision at all is made for equalizing the brightnesses of the two fields; hence, the problem, strictly a tridimensional one, is reduced to a single specification by neglecting both small chromaticity and small brilliance differences.

VI. Summary

The uniform-chromaticity-scale triangle is shown to be in good agreement with a large proportion of published data on chromaticity sensibility. The scope, reliability, and consistency of these data are such that it is planned to use the triangle for estimation of chromaticity differences and for the determination of nearest chromaticity from a series for any neighboring chromaticity.

VII. Appendix

If it be desired to avoid the use of triangular coordinate plots, the trilinear coordinates, r, g, b, may be plotted in rectangular coordinates by the transformation:

$$\begin{aligned} x &= (2b+g)/3^{\frac{1}{2}}, \\ y &= g, \end{aligned} \tag{11}$$

where x and y are abscissa and ordinate, respectively, of the usual rectangular coordinates.[39] These transformations place the equilateral triangle (see Fig. 1) in the first quadrant of the (x, y)-plot, with the G-point $(r=0, g=1, b=0)$ uppermost and the R-point $(r=1, g=b=0)$ at the origin $(x=y=0)$.

In rectangular coordinates relation (2) may be written in the familiar form:

$$D=(\overline{\Delta x}^2+\overline{\Delta y}^2)^{\frac{1}{2}} \tag{12}$$

and relation (2a) becomes:

$$D=\begin{cases}\Delta x[(\Delta y/\Delta x)^2+1]^{\frac{1}{2}} & \text{or} \\ \Delta y[(\Delta x/\Delta y)^2+1]^{\frac{1}{2}}. \end{cases} \tag{12a}$$

[38] R. Davis, *A Correlated Color Temperature for Illuminants*, Bur. Standards J. Research **7**, 659 (1931); RP365.

[39] Note distinction from $\bar{x}$ and $\bar{y}$ of relation (3).

Photometry and Energy Distribution of the High Intensity Mercury Lamp

L. B. JOHNSON, *General Electric Vapor Lamp Company*

(Received August 7, 1934)

THE problem of photometry of the high intensity mercury lamp is one of color difference between standard and lamp, so often met with in gaseous conduction light sources.

The high intensity mercury lamp is a light source made thermally stable by an insulating enclosure containing nitrogen at one-half an atmosphere pressure, and by the use of a limited amount of mercury such as to insure complete vaporization at normal operating temperature.

The usual method is to use a vacuum tungsten filament lamp of known light output at some known color temperature, as a standard. By the use of filters of known transmission between the standard and the photometer head, a color match can be approximated. The light output of the standard is corrected for the transmission of the filter. The object of using filters is to produce a color match (an energy match, if possible) so that the observer should obtain the same results as a hypothetical observer who could compare accurately two lamps differing greatly in color.

Buckley[1] has described methods of photometry of gaseous conduction lamps using an incandescent standard with appropriate filters.

An alternative method is to compare the high intensity lamp with some other mercury lamp used as a standard, such as a 50 inch low pressure mercury lamp. The candle power of the 50 inch low pressure lamp has been found by spatial distribution measurements to be approximately 687 cp. Measurements have been made on these lamps by the Electrical Testing Laboratories and Mr. F. Benford[2] of the General Electric Research Laboratories. Subsequent comparisons have found the regular production lamps very uniform in light output.

The low pressure lamp and the high intensity lamp emit the same kind of light, that is, mercury spectral lines, the only difference being a different distribution of the line intensities. A method of photometry which would compare only individual lines, would give an energy match, and avoid errors due to subjective color differences still apparent when filters are used with an incandescent standard.

The visible light from both mercury lamps is due essentially to yellow, green and blue lines; the red and violet lines, although contributing considerable energy, add a negligible amount of light because of the low relative visibility.

The luminosities produced by the high intensity lamp and those produced by the low pressure lamp may be compared for the individual lines, by placing "monochromatic" filters between the eye and eyepiece of the photometer head and matching the resulting "monochromatic" fields. A bench photometer is used, with a Lummer-Brodhun photometer head, with a contrast field. The variable distance method is used, where the lamps are stationary.

The "monochromatic" filters transmit the following lines:

Corning 348	5770–5791 and red
(2) Corning 512 and (2) Corning 351	5461 and red
Wratten no. 50	4358–4348–4339

The actual percentage transmission of these filters is irrelevant since the lines transmitted are the same for both sources and thus reduced by the same percentage. The 4916 line which has a rather high relative visibility is very weak and is not transmitted by the filters. The red transmitted by the green and yellow filters may be neglected because of the low relative visibility and the low transmission of the filter.

Luminosities are matched by the photometer, but they are essentially monochromatic and it is comparable to matching energies, since the relative visibilities are identical. It is really a method of spectrophotometry.[3] Thus the best possible condition for photometry is obtained, where the color match has become an energy match.

[1] H. Buckley, Illuminating Engineer (London) **27**, 118, 148 (1934).

[2] F. A. Benford, Trans. Motion Pict. Eng. **30**, 305 (1927).

[3] J. W. T. Walsh, *Photometry*, p. 288, D. Van Nostrand Co.

The candle power of the standard may be divided into three groups, yellow, green and blue, if the energies and relative visibilities for the groups are known.

The relative line intensities of the low pressure lamp were unknown and had to be determined by comparing "monochromatically" two of the low pressure lamps operated at 3.7 amperes and side by side, with a 3 inch Uviarc operated at 80 volts and 4 amperes. (Two low pressure lamps were used to obtain almost equal photometric field intensities.) The energy distribution of a 3 inch Uviarc and of a 22 inch low pressure quartz arc have been reported in a paper by Johnson and Burns.[4] B. T. Barnes[5] has reported line intensities of the 3 inch Uviarc, and 50 inch glass low pressure lamp, measured with a calibrated double monochromator. These measurements compare favorably with those of Johnson and Burns. The relative line intensities found for the two low pressure lamps is the same as that of the 22 inch quartz lamp, as might be expected, and checks well with the results given by Barnes.

The photometric measurement of the candle power of the high intensity lamp is made by comparing a high intensity lamp by the method outlined above with a mercury standard composed of two 50 inch low pressure lamps, operated side by side at 3.7 amperes. (Two lamps are used to obtain nearly equal photometric field intensities.) The candle power of the standard is then equal to the sum of the candle power of the two low pressure mercury lamps, 1374 candle power.

The comparison of the high intensity lamp and the low pressure standard results in a ratio of the individual line intensities of the high intensity lamp to those of the low pressure lamp.

The percentage of the candle power in the yellow region of low pressure lamps is represented by the product of the percent of the total energy in the yellow and the relative visibility for the group, divided by the sum of the respective products of the percentage of the yellow, green and blue groups and their relative visibilities.

The candle power of the yellow group for the high intensity lamp is, for example, the product of the total of yellow light in the low pressure lamps and the ratio of the yellow energy in the high intensity lamp to that in the low pressure lamps, found by comparing the high intensity lamp against the low pressure standard.

The following is an example of the method described above. The distribution in the two low pressure lamps found by comparison with the Uviarc is

Group	*Energy distribution*	*Relative visibility*
Yellow	14.8 percent	0.890
Green	43.3	0.990
Blue	41.9	0.020

The percent of the candle power for each group is

Yellow	23.1 percent
Green	75.4
Blue	1.5

The ratios of the intensities of the high intensity lamp to those of the low pressure lamps are

Yellow	2.24
Green	0.690
Blue	0.536

The total light output of the two low pressure lamps is 1374 cp. The light output of the high intensity lamps is then

Yellow	$1374\times2.24\times23.1/100=$	711
Green	$1374\times0.690\times75.4/100=$	715
Blue	$1374\times0.536\times1.5/100=$	11
		1437 cp.

The ratio of candle power taken with the lamp rotating about its vertical axis (mean horizontal candle power) to that taken at right angles to the lead wires (beam candle power) is 0.962.

The factor for converting mean horizontal candle power to lumens is 10.36.

The lumen output of the high intensity lamp is then $1437\times0.962\times10.36=14{,}300$ lumens.

The above data were for one lamp of a group of five high intensity mercury lamps which were measured by the "monochromatic" method of

[4] L. B. Johnson and L. Burns, J. O. S. A. **23**, 55 (1933).
[5] B. T. Barnes, J. O. S. A. **24**, 147 (1934).

photometry. The average for the five lamps is

Lamp watts	Lamp current	Lamp volts	Lumens
397	2.90	149	14,200

TABLE I.

Lamp	Energy distribution (%) Yellow	Green	Blue	Lumen distribution (%) Yellow	Green	Blue
Average 5 high intensity lamps	39.8	34.8	25.4	50.3	49.0	0.7
*3 inch Uviarc	40.2	33.0	26.8	52.0	47.3	0.7
50 inch low pressure lamp	14.8	43.3	41.9	23.1	75.4	1.5

* Operated at 80 volts and 4 amperes.

I am indebted to Dr. Little of the Electrical Testing Laboratories for making spatial distribution and lumen measurements on the high intensity lamp and the low pressure lamp. These measurements made possible the calculation of lumens from beam candle power, as well as serving as a check on the "monochromatic" method, since the Electrical Testing Laboratories results were in good accord with those of the writer. The candle power assigned by them, to the low pressure standard, was within one percent of the value assumed when the lamps were removed from stock for use as mercury standards.

One of the merits of the "monochromatic" method is, that once the mercury standard has been established, it may be used for photometry of other mercury lamps differing in color from the high intensity lamp because of different distribution of line intensities. Also of interest to the lamp manufacturers is the lumen distribution which is obtained in the course of the photometry by the "monochromatic" method, because he is as much interested in a lamp's color as he is in its light output and efficiency.

Notice

THE Photographic Society of America which is the recent expansion program of the Associated Camera Clubs to extend service to those interested in photography, whether they be members of clubs or not, will begin its second year by publishing its bulletin in printed form with a few excellent illustrations. The society is also planning a national meeting at Pittsburgh in April of 1935 in conjunction with the Pittsburgh Salon of Photography. Those interested in the work of this organization may address its secretary, B. H. Chatto, 1300 Milton Avenue, Pittsburgh, Pennsylvania.

A Study, by Means of Huygen's Principle, of the Reflection of a Spherical Light Wave from a Moving Plane Mirror

NADIASHDA GALLI-SHOHAT,* *Rockford College, Rockford, Ill.*

(Received November 1, 1934)

LET S in Fig. 1 be a point in space where at a certain instant a single spherical light wave was emitted. Consider the case when the mirror moves *toward* the incident wave, in which case the angle θ which the direction of motion v makes with the normal to the mirror is an acute angle. Let MM be the *initial* position of the mirror at the moment when the spherical wave is emitted, M_1M_1—its position when the wave meets it at the point N_1 and M_2M_2—its position when a ray of the angle of incidence $\varphi = N_1SO$ meets it at the point O.

When the mirror advances in the direction of the arrow, Fig. 1, with velocity v, it advances toward S along its normal with velocity $v_N = v\cos\theta$. Denoting v_N/c by β_N (c is the velocity of light), we may write:

$$NN_1 = \beta_N SN_1, \quad NN_2 = \beta_N SO,$$
$$N_1N_2 = \beta_N(SO - SN_1). \quad (1)$$

We proceed to find N_1N_2. From the triangle SON_2:

$$SO = SN_2/\cos\varphi = (SN_1 - N_1N_2)/\cos\varphi. \quad (2)$$

Substituting this in N_1N_2 (1) and solving for N_1N_2, we have,

$$N_1N_2 = \beta_N p(1-\cos\varphi)/(\cos\varphi + \beta_N), \quad (3)$$

where $p = SN_1$. This gives the position of the mirror, where all rays of the angle of incidence φ, forming a cone, meet the moving mirror. One can see (by differentiation) that N_1N_2 increases with the angle of incidence φ, at first slowly, then more rapidly, so that the *locus* of the points in space where individual rays meet the moving mirror is a concave surface. If we construct around these points, according to Huygen's principle, the elementary waves and their envelope, we can see that reflection actually occurs as if from a concave mirror *HH fixed in space* and passing through the points N_1 and O, Fig. 1. We wish first to find the image of S in a moving mirror and the shape and dimensions of the fixed concave mirror.

In order to find the image of S in a moving mirror we shall find the point of intersection of the reflected rays. A sufficiently small part of a spherical wave may be considered as a plane wave; thus, in order to construct the reflected rays we can use the law of reflection of a plane wave from a moving plane mirror. This law, derived by means of Huygen's principle, has been given in various forms by different authors. The most convenient form for our purpose is that given by E. Kohl:[1]

$$\varphi_r = \varphi - 2\beta_N \sin\varphi + 2\beta_N^2 \sin\varphi\cos\varphi, \quad (4)$$

where φ and φ_r are the angles of incidence and reflection, respectively, and where the terms of order higher than the second are neglected. We shall write (4) in the form

$$\varphi_r = \varphi - 2\alpha,$$
$$\alpha = \beta_N \sin\varphi - \beta_N^2 \sin\varphi\cos\varphi. \quad (5)$$

Here α is the angle which a *fixed* plane mirror makes with the moving plane mirror.

We now apply the above law to the incident ray, SO. In order to carry out the actual construction of the reflected ray, we first neglect terms of the second order in β_N and find the reflected ray in the first approximation. Later we introduce a correction of the second order.

Let P and P_1, Fig. 1, be the virtual images of S in the mirrors MM and M_2M_2, respectively, if they were at rest. Thus, SO will be reflected from M_2M_2 in the direction P_1Q_1, so that we may write:

$$\angle DOQ_1 = \varphi, \quad PN = SN, \quad P_1O = SO,$$
$$PP_1 = 2NN_2. \quad (6)$$

* This work was done, to a great extent, while the author was a holder of the Helen Schaeffer Huff Memorial Fellowship in the Department of Physics, Bryn Mawr College.

[1] E. Kohl, Ann. d. Physik **28**, 259 (1909).

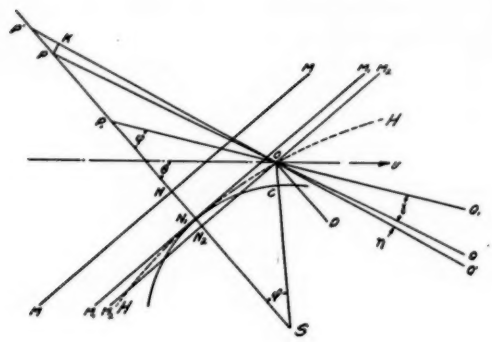

FIG. 1.

Draw a line PQ through O and denote $<QOQ_1 = <POP_1$ by γ. We wish to show—and this is the cardinal point in our discussion—that if we neglect terms of the second order in β_N, then

$$\gamma = 2\alpha. \tag{7}$$

In fact, from the triangle POP_1 [see also (6) and (1)]

$$\sin\gamma/\sin(\varphi-\gamma) = PP_1/P_1O = 2NN_2/SO = 2\beta_N. \tag{8}$$

From this, neglecting terms of the third order in β_N, inasmuch as $\tan\gamma$ is of the first order in β_N and $\gamma-\tan\gamma$ is of the third order in $\tan\gamma$, we may write:

$$\begin{aligned}\tan\gamma &= 2\beta_N\sin\varphi/(1+2\beta_N\cos\varphi)\\ &= 2\beta_N\sin\varphi(1-2\beta_N\cos\varphi),\\ \gamma = \tan\gamma &= 2\beta_N\sin\varphi - 4\beta_N^2\sin\varphi\cos\varphi.\end{aligned} \tag{9}$$

Combining this with (5), we get:

$$\eta = 2\alpha-\gamma = 2\beta_N^2\sin\varphi\cos\varphi. \tag{10}$$

Hence, neglecting β_N^2, we obtained the result as stated above in (7).

This holds true for all the angles of incidence; hence, all rays, and consequently the whole wave, appear to come from the point P—the image of S in the concave mirror—which at the same time is the image of S in the plane mirror MM, as if the mirror were at rest, and the wave were reflected directly from MM.

We are now in a position to show that the fixed concave mirror is a hyperbolic mirror. From the triangle POP_1:

$$PO/P_1O = \sin\varphi/\sin(\varphi-\gamma). \tag{11}$$

According to (8):

$$\sin(\varphi-\gamma) = \sin\gamma/2\beta_N.$$

If we neglect terms of the second order in β_N, we can write (9), since $\gamma-\sin\gamma$ is of the third order in γ as follows:

$$\sin\gamma = \gamma = 2\beta_N\sin\varphi(1-2\beta_N\cos\varphi),$$

substituting into (11), and since $P_1O = SO$ and making use of (2) and (3), we get finally:

$$PO-SO = 2\beta_N\left(p-\beta_N\frac{1-\cos\varphi}{\cos\varphi+\beta_N}\right). \tag{12}$$

For all the angles of incidence, except those for which $\cos\varphi$ becomes of the order β_N, the second term in (12) can be neglected. Thus,

$$PO-SO = 2\beta_N p = \text{constant},$$

which shows that the point O lies on a hyperbola with S and P as foci, and the mirror therefore is a *hyperbolic* one. When φ increases further, $PO-SO$ becomes continuously smaller and in the outer zone the mirror becomes flatter.

We now take into consideration terms of the second order, and introduce the corresponding correction in the construction of the reflected ray.

Since here, by (10), $2\alpha>\gamma$, we must turn the ray OQ in Fig. 1 still further toward the normal OD by the angle $\eta = <QOQ'$. OQ' will be the corrected reflected ray, which now appears to come from the point P'.

From the triangle POP', by (8) and (10),

$$\begin{aligned}PP'/P'O &= \sin\eta/\sin(\varphi-\gamma) = 2\beta_N^2\cos\varphi,\\ PP' &= 2\beta_N^2 P'O\cos\varphi = 2\beta_N^2 PO\cos\varphi.\end{aligned} \tag{13}$$

We replaced here $P'O$ by PO, for they differ in the second order which affects PP' in the fourth order.[2]

From the right triangle PON_2, Fig. 1, we have further:

[2] See Fig. 1, triangle $PP'K$, where $PK = P'O-PO$ is of the order of PK, that of the angle η, by (10).

$PO=PN_2/\cos(\varphi-\gamma)=(PN_1+N_1N_2)/\cos(\varphi-\gamma)$.

Replacing here $\cos(\varphi-\gamma)$ by $[1-\sin^2(\varphi-\gamma)]^{\frac{1}{2}}$, as derived from (8), we get:

$$PP'=\frac{2\beta_N^2(PN_1+N_1N_2)}{1-2\beta\tan\varphi\cos\varphi+\cdots}=2\beta_N^2(q+N_1N_2).$$

Here terms of the third order in β_N are neglected, and $q=PN_1$. Substituting N_1N_2 from (3),

$$PP'=2\beta_N^2q\left(1+\beta_N\frac{p}{q}\frac{1-\cos\varphi}{\cos\varphi+\beta_N}\right). \quad (14)$$

For all the angles of incidence specified above, i.e., for which the second term in (14) could be neglected, we thus have:

$$PP'=2\beta_N^2q=\text{const.} \quad (15)$$

This gives P' as a corrected image of S in the concave mirror. As φ increases further, PP' also increases and there will be aberration.

From the object-distance $p=p_0(1-\beta_N+\beta_N^2)$ and the image-distance $P'N_1=q+PP'=p_0(1+\beta_N+\beta_N^2)$, where $p_0=SN$ is the distance of S from the initial position of the mirror, we get for the focal distance F and the radius of curvature R of the concave mirror:

$$F=p_0(1+\beta_N^2)/2\beta_N, \quad R=p_0(1+\beta_N^2)/\beta_N. \quad (16)$$

It follows that the curvature of the mirror depends on at what distance from the mirror the spherical wave was emitted, and upon the orientation of the mirror with respect to the motion (angle θ). In the case when the mirror moves along its plane ($\theta=90°$, $\beta_N=0$, $R=\infty$), it remains plane. The curvature of the mirror is maximum when it moves perpendicular to its plane ($\theta=0$, $\beta_N=\beta$). In the latter case if we take, according to modern astronomical data, the velocity of the solar system in space $v=300$ km/sec., thus $\beta=10^{-3}$, we get, for $p_0=1$ meter, a concave mirror with the radius of curvature of *one kilometer*.

If the mirror moves *away* from S we must replace in our formulae β_N by $-\beta_N$, and the fixed mirror is in this case *convex*.

Proceedings of the Nineteenth Annual Meeting of the Optical Society of America

NATIONAL BUREAU OF STANDARDS, WASHINGTON, D. C.

October 18–20, 1934

THE nineteenth annual meeting of the Optical Society of America was held at the National Bureau of Standards, Washington, D. C., October 18 to 20, 1934.

The meeting was opened Thursday, October 18, at 9.30 A.M. by a welcome from Dr. Lyman J. Briggs, Director of the Bureau.

SESSION DEVOTED TO OPTICS OF ASTRONOMICAL INSTRUMENTS

The session Thursday afternoon was devoted to papers on "Optics of Astronomical Instruments." The following papers were presented by invitation:

The Influence of Present Trends in Research on the Design of Astronomical Telescopes and Accessories, by G. W. Moffitt, Yerkes Observatory

New Astrophysical Instruments of the Harvard Observatory, by F. L. Whipple and D. H. Menzel, Harvard College Observatory

Applications of Christianson Filters to Stellar Energy Spectra, by C. G. Abbot, Astrophysical Laboratory, Smithsonian Institution

Optics of Reflecting Telescopes, by Frank E. Ross, Yerkes Observatory

The Work of the Naval Observatory, by J. F. Hellweg, U. S. Naval Observatory

The Ritchey-Chrétien 40-inch Aplanatic Reflector—Principles of Design and Practical Results, by G. W. Ritchey, U. S. Naval Observatory.

VISIT TO THE U. S. NAVAL OBSERVATORY

On Thursday evening the Naval Observatory held open house for members of the Society and their guests. Opportunity was afforded for inspecting the instruments and looking through the large telescopes. The principal object of interest was the new Ritchey-Chrétien 40-inch reflector.

ANNUAL DINNER

Because of unfavorable weather the annual dinner of the Society could not be held on the lawn as planned but was held instead in the Hydraulic Laboratory of the National Bureau of Standards at 5.00 P.M., Friday, October 19. There was no formal program but the members found it a pleasant opportunity for visiting and informal discussions.

PUBLIC LECTURES

On Friday evening at 7.30 P.M. two public lectures were given. Mr. Charles Bittinger, Washington, D. C., gave a short talk on "Ultraviolet Murals at the Franklin Institute," illustrating the talk by painting a sketch with ultraviolet pigment under the illumination of a source of ultraviolet light.

This was followed by a lecture by Dr. F. G. Pease of Mount Wilson Observatory on "Modern Large Telescope Design." Dr. Pease gave an interesting discussion of the many problems involved in the design of the 200-inch telescope now in process of construction.

BUSINESS SESSION

A brief business session of the Society was held Saturday morning, October 20, President Rayton presiding. The Secretary and Treasurer laid before the Society the formal reports for the calendar year 1933* and gave informal statements of the changes which had taken place since January 1, 1934. The reports were accepted.

The Secretary was instructed to write to the National Bureau of Standards and to the U. S. Naval Observatory expressing the thanks of the Society for the hospitality which it had enjoyed.

* Published in the J. O. S. A. **24**, 308–311 (1934).

EXHIBITION OF INSTRUMENTS

The exhibition of instruments and products in which the application of optical principles plays an important part in design, construction, or use, attracted much attention. Beside the fine exhibits of commercial apparatus there were exhibits from research laboratories showing new experiments and recent developments in apparatus.

REGISTRATION

Members 73, Guests 15, Total 88.

SESSIONS FOR THE READING AND DISCUSSION OF CONTRIBUTED PAPERS

Sessions for the reading and discussion of contributed papers were held as follows:

Thursday, October 18, at 9.30 A.M.
Friday, October 19, at 9.30 A.M.
Friday, October 19, at 1.00 P.M.
Saturday, October 20, at 10.00 A.M.

The authors' abstracts of these papers are appended.

MEETING OF THE BOARD OF DIRECTORS

Sessions of the Board of Directors were held on Wednesday afternoon and evening, October 17.

L. B. TUCKERMAN, *Secretary.*

Minutes of the Fourth Meeting of the Directors of the Optical Society of America, Incorporated

WASHINGTON, D. C.
October 17, 1934

BESIDES the transaction of routine business, the following items of general interest to members of the Society were taken up.

Use of Coated Paper

It was voted that commencing with January 1, 1935, the Editor should arrange for the use of a satisfactory coated paper for the Journal so as to facilitate the use of photographic illustrations in connection with articles.

List of Members to be Issued

It was voted that in or with the March, 1935 issue there should be published a list of members and such other material as might be prepared by a committee appointed by the President, the committee to have power to decide the manner of issue and the number of copies.

The Treasurer noted that the estimated extra cost involved in using coated paper and in issuing the membership list would still allow a balanced budget.

Report of Representative on American National Committee on International Congresses on Photography

Professor A. C. Hardy, representative of the Optical Society of America on the American National Committee on International Congresses on Photography, reported that the committee had in active preparation its report for the meeting to be held in Paris, July 7–13, 1935.

Report of Representative on U. S. National Committee of the International Commission on Illumination

In the absence of Mr. Frank Benford, representative of the Optical Society on the U. S. National Committee of the International Commission on Illumination, Mr. E. C. Crittenden reported that the committee would meet November 9 to prepare for the meeting of the Commission in Berlin, July 2–9, 1935.

Ives Medal

It was noted that in accordance with the terms of the foundation the Ives Medal would be awarded again in 1935.

Transfer from Associate to Regular Membership

The following were transferred from associate to regular membership:

Carl E. Bahn (942 R)
J. A. Ball (964 R)
Vola Price Barton (757 R)
Julian M. Blair (948 R)
F. Chapin Breckenridge (594 R)
Brooks A. Brice (882 R)
Laurence Burns (963 R)
Gordon A. Chambers (474 R)
J. H. Ellis (494 R)
Gustave Fassin (943 R)
David L. Gamble (950 R)
E. O. Hulburt (970 R)
David L. MacAdam (930 R)
Gjon Mili (952 R)
Cyril J. Staud (961 R)

L. B. TUCKERMAN, *Secretary.*

Program of Sessions for Contributed Papers*

(Titles and Abstracts of Papers)

1. Surface Color. Deane B. Judd, *National Bureau of Standards.* (20 minutes.)

Their tristimulus specifications are sufficient for deciding if the colors of two surfaces are the same; but, since the visual mechanism has widely varying sensitivity depending on simultaneous stimulation of adjacent areas and on previous general stimulation, the tristimulus specification of a surface is not sufficient to determine its color. The present work deals with the usual case of the observer well adapted to the illuminant. The aim is to determine surface color in terms of hue, brilliance and saturation from tristimulus specifications by including also specifications of the background.

The surfaces used were 15 papers, roughly inch-square, of known spectral reflectance. The 5 illuminants were daylight and 4 chromatic illuminants, each of the latter consisting of an incandescent lamp combined with a highly selective glass filter of known spectral transmission. The two backgrounds used were dark gray and white cardboard. Estimates of hue, brilliance, and saturation made by 8 observers on each of the 150 colors were correlated with tristimulus specifications computed from the known spectral energy distributions. It was found that brilliance could be satisfactorily represented by the formula due to Adams and Cobb:[1] Brilliance $=aR/(R+R')+b$, R and R' being the reflectances of sample and background, respectively, and a and b, constants. It was found that saturation of the colors of papers having nearly the same reflectance as the background is roughly proportional to distance on the uniform-scale Maxwell triangle[2] between the point representing the sample color and that representing the background color; that is, for such papers the background color usually is taken as of zero saturation. For such papers also, hue could be correlated closely with the direction of the line connecting the two points. Colors notably lighter than the background assumed somewhat the hue of the illuminant while those much darker than the background tended toward the complementary.

It seems possible from preliminary trials to write empiric formulas giving hue and saturation in quantitative accord with these results, but successful formulas have not yet been found.

[1] J. Exp. Psych. **5**, 39 (1922); J. O. S. A. and R. S. I. **9**, 932 (1922).
[2] J. O. S. A. **24**, 163 (1934).

2. Color Blending Computations in Psychological Terms. I. H. Godlove, *Color Service Laboratories, Washington, D. C.* (15 minutes.)

The tristimulus specifications of Munsell colors afford data for testing various rules for computation of hue and saturation of blends obtained by rotation directly from psychological specifications of the components. If we assume only that component colors and their blend may be represented in Euclidian cylindrical coordinates and that Newton's and Talbot's laws are valid, we obtain for the case of two hues separated by one-fifth of the hue circuit

$$\tan\beta=\frac{0.951\,S_2}{0.309\,S_2+vS_1} \quad \text{and} \quad S=\frac{(n^2+1.618)^{\frac{1}{2}}}{1+v}\cdot S_2$$

where β is the angle between the vertical planes through the points representing the first color and the blend, S is the saturation of the blend, S_1 and S_2 the saturations of the components,

$$v=(V_2-V)/(V-V_1), \qquad n=vS_1/S_2$$

and the V's are the brilliances of the 2 components and the blend. Making some simple approximations, and using a 100-step hue scale numbered from the hue of the first component, we obtain

$$H=\frac{vS_1H_1+S_2H_2}{vS_1+S_2}.$$

Alternatively, if we assume that hue-number is the weighted mean of the hue-numbers of the components, and similarly for saturation, the weighting factors W and w being, respectively, $A\cdot V\cdot S$ and $A\cdot V$, where A is the fractional area, then we obtain

$$H'=\frac{W_1H_1+W_2H_2}{W_1+W_2} \quad \text{and} \quad S'=\frac{w_1S_1+w_2S_2}{w_1+w_2}.$$

Formulas for H and S have been published by Miss Nickerson and used by others. Her hue formula and the formula for H' are mathematically equivalent to the approximate one for H for the case of constant brilliance, though they have been used for combinations of different brilliances; her formula for saturation is the reduction form of S' at constant brilliance, and of S at constant hue and brilliance, though used for the general case, becoming absurd when the components are complementary.

On interpreting the computed specifications of 23 blends of 2 components each to Munsell notations, it is found that the Nickerson hue formula does not agree with the data quite as well as the other formulas; and her saturation formula does not agree quite as well as the formula for S, for large hue range, but for small hue ranges gives about equal agreement. Since the errors are greater than the probable uncertainty in the data, it is concluded that the local irregularities in the Munsell data are such as not to justify the use of any of the formulas for large hue ranges.

The present paper will appear in full in J. O. S. A.

* The abstracts were preprinted in the program subject to correction by the authors. Corrections that have been communicated to the secretary have been made in the abstracts reprinted below.

3. Aviation Lighting Research at the National Bureau of Standards. (Illustrated.) F. C. BRECKENRIDGE, *National Bureau of Standards.* (20 minutes.)

Aviation lighting research is carried on at the National Bureau of Standards for the Bureau of Air Commerce, Commerce Department and Bureau of Aeronautics, Navy Department. Typical illustrations of the development of aircraft landing lights and position lights will be shown and methods of beacon photometry will be described. An outline of present projects will also be given.

4. Determining the Difficulty of Visual Tasks. M. LUCKIESH AND F. K. MOSS, *Lighting Research Laboratory, General Electric Company.* (20 minutes.)

The importance of seeing emphasizes the necessity of devising means for determining the difficulty of visual tasks. In recent years the problem has been attacked through elaborate and tedious laboratory researches. The results are valuable in helping to lay the foundation for a science of seeing, but such methods are not practicable for general application. However, they seem to lead to correlations which are helpful in the development of simple devices which may easily be used. Such devices may involve the determination of threshold visibility which in itself is not necessarily a measure of the difficulty of seeing, but by establishing correlations between this factor and ease of seeing, such measurements become practicable in appraising at least the relative difficulty of a series of visual tasks. The general principles involved and some simple devices will be discussed. Particular emphasis will be placed upon a visual thresholdometer and results obtained with it.

Bibliography:

A brief paper entitled "A Visual Thresholdometer" has been transmitted to the J. O. S. A.

5. A Convenient Set of Equations for Computing Third Order Aberrations. I. C. GARDNER, *National Bureau of Standards.* (15 minutes.)

The Seidel equations for the third order aberrations of the surfaces of a lens system, composed of thick lenses, have been transformed into a set of equations particularly convenient for logarithmic computation and especially adapted for use in conjunction with trigonometric computations for the separation of the third order and the higher order aberrations.

6. A Graphical Method of Locating Conjugate Image Points. HAROLD F. BENNETT, *Optical Shop, Washington Navy Yard.* (10 minutes.)

A graphical method of locating conjugate points of a lens is demonstrated. This method, while not new except for minor modifications, seems to merit a much wider use than it now has. It presents a simple geometrical figure by the aid of which the relative positions of object and image may be easily followed as either is moved through any range of positions from plus to minus infinity. It may be applied to any lens or mirror system for which the principal points and principal foci may be located. It is expected to prove useful in planning optical systems and also in acquainting students with the applications of the Gaussian lens equations.

7. The Spectrographic Program of the National Geographic-U. S. Army Stratosphere Flight. BRIAN O'BRIEN AND T. A. RUSSELL, *Institute of Optics, University of Rochester, N. Y.* (20 minutes.)

Determinations of the height and distribution of ozone in the earth's atmosphere, based upon measurements of the ultraviolet solar spectrum as received at the earth's surface, indicate the center of gravity of the ozone layer at an elevation of 30 to 50 kilometers, and little or no ozone below 10 km. It has appeared probable that ozone exists in the region between 10 and 20 km above the earth's surface in amounts which would be measurable providing sufficiently refined instruments could be carried to these heights. The present balloon, with its great weight carrying capacity and an estimated ceiling well over 20 km, provided an unusual opportunity for making such measurements.

Estimation of atmospheric ozone by intensity measurements within and just outside the long wavelength end of the Hartley band has proven very successful at ground stations and was adopted here. Two quartz spectrographs of medium dispersion, equipped with evaporated aluminum wedges for the measurement of intensity, were provided for this purpose. These received, respectively, direct sunlight and skylight 8° from the horizon, the latter providing a more sensitive measure of the lower limit of the ozone layer.

At wavelengths shorter than the Hartley band of ozone the solar spectrum as received at the earth's surface is limited by the absorption of oxygen. At an elevation of 20 km with the partial pressure of oxygen less than 6 percent of that at sea-level, the possibility exists of photographing the solar spectrum in the region around 2100A between the oxygen and ozone bands. A third quartz spectrograph was provided for this purpose, intensities being measured as in the larger instruments by means of an evaporated aluminum wedge. Special provisions were made to eliminate scattered light of longer wave-length. Quartz was selected for high transmission to 1850A, and the sensitivity of the instrument was much increased by the use of an Eastman type I–0 emulsion coated with the fluorescent collidine derivative described by Mees.

8. Stratosphere Spectrographs. H. F. KURTZ AND GUSTAVE FASSIN, *Bausch & Lomb Optical Company.* (20 minutes.)

The three spectrographs built for the U. S. Army-National Geographic Stratosphere Flight will be described. The purposes for which each was intended will be set forth. The optical systems will be described. The mechanical construction, including the details of the automatic operating mechanism, will be discussed.

9. A Testing Camera for Photographic Objectives. I. C. GARDNER AND F. A. CASE, *National Bureau of Standards.* (10 minutes.)

A new camera for testing photographic objectives will be described. This apparatus has been specially designed in order to facilitate the tests of the large number of airplane camera lenses which are sent to the National Bureau for test. With this equipment a negative is obtained on which

are registered 100 or more individual exposures made of a test chart and, from this negative, fairly complete information can be obtained regarding all the aberrations of the photographic objective. The results of some of these tests will be shown.

10. The Fifty-Inch Precision Copying Camera of the U. S. Coast and Geodetic Survey. O. S. READING, *U. S. Coast and Geodetic Survey.* (10 minutes.)

Repeated revision surveys of the coast and harbors of this country require photographic equipment capable of reproducing the new surveys accurately enough to fit the old without expensive and slow adjustment. During the past year the largest precision copying camera in the world has been completed to accomplish this work for the U. S. Coast and Geodetic Survey. The adjustments and precautions used to attain precision of one or two thousandths of an inch in photographs made with the camera are briefly described.

11. A Proposal in Nomenclature. CLAYTON H. SHARP, *Consulting Engineer, White Plains, N. Y.* (5 minutes.)

The name given to any new scientific device should be distinctive and, if possible, descriptive. A good name should be given in early infancy, before one less desirable has become established. In the nomenclature of photoelectric devices the name "photoelectric cell" is by common consent reserved for the alkali metal, vacuum cell. "Photo-resistance cell" and "photo-voltaic cell" are acceptable descriptive names for their respective classes. The designations given to the newest class of cells, "barrage" or "barrier-layer" cells, are neither distinctive nor descriptive and give no intimation that they are photoelectric devices at all. The name "photo-e.m.f. cell" is proposed for criticism and discussion. The objections which readily occur to this name do not seem to be very weighty in view of the aptness of the term and the convenience which would result from its use.

12. A Filter Isolating 560 mμ. KASSON S. GIBSON, *National Bureau of Standards.* (10 minutes.)

A filter freely transmitting a narrow spectral region centering at 560 mμ and strongly absorbing at all other wave-lengths in the visible spectrum is of use in certain problems in (1) the colorimetry of sugar solutions,[1] (2) optical pyrometry,[2] (3) abridged spectrophotometry,[3] and (4) photometry.[4] A filter has recently been designed having the following components:

Designation of glass	Thickness in mm
1. Corning 351	4.55
2. Corning didymium	5.82
3. Jena VG 3	1.99
4. Jena BG 18	1.94

These are cemented together with Canada balsam.

This 4-component filter has the following characteristics:

Wave-length (mμ)	Transmission	Wave-length (mμ)	Transmission
530	0.001	560	0.275
540	.002	565	.212
545	.003	570	.010
550	.028	520 and less	less than .001
555	.174	575 and greater	

The spectral centroid of the transmitted light varies from $560._0$ mμ for I.C.I. illuminant *A* (incandescent light, 2848° K) to $559._8$ mμ for I.C.I. illuminant *C* (average daylight). The luminous transmission is 0.03_2 for illuminant *A* and 0.03_5 for illuminant *C*.

The spectral transmissions of the filter and its components will be illustrated and comparisons made with other similar filters. The transmission curve of the present filter has a ratio of height to width more than twice as great as the similar ratios for the other filters, including the one by Livingston[4] in which three of the four components of the present filter were used, although at different thicknesses. The present filter had been constructed before his description was published.

[1] Peters and Phelps, Techn. Pap. Bur. Standards **21**, 261 (1927); Bur. Standards J. Research **2**, 335 (1929); Brewster, Bur. Standards J. Research (in press).
[2] Hottell and Broughton, Ind. Eng. Chem. **4**, 166 (1932).
[3] Appel, Am. Dyestuff Reporter **17**, 49 (1928).
[4] Livingston, J. O. S. A. **24**, 227 (1934).

13. Radiometric Measurements of Heat Absorbing Glass. ROGER S. ESTEY, *Electrical Testing Laboratories, New York, N. Y.* (10 minutes.)

As a class, heat absorbing glasses include all those which are relatively more transparent to visible than to invisible radiations. They are used in various applications for reducing the heat received from light sources. Special glasses used in windows to keep out solar heat are of present commercial importance and their testing is therefore of scientific interest. The effectiveness of a glass in a particular application can be determined theoretically by comparing the spectral transmission curve of the glass with the spectral energy distribution of the source. Practically, the light and heat transmission are each determined by single measurements using the source for which the sample is designed. In the case of the sun the infrared transmission of the atmosphere is too variable to permit close comparisons between measurements made at different times and good weather is too uncertain for convenience. Unmodified tungsten lamp radiation has been substituted for the sun as a test source to some extent. The differences between lamp and sun measurements are large and variable depending on glass color and thickness. If the tungsten light is modified by a daylight filter and if the tungsten radiation is modified by a filter producing an energy distribution approximately like that of solar radiation a laboratory source is produced which yields transmission values in much closer agreement with solar measurements. Data are presented showing the degree of agreement in light and heat transmission measurements made on various heat absorbing glasses using the sun and laboratory sources.

14. The Scattering of Light by Suspensions of Cells. HAROLD MESTRE, *Yale University.* (20 minutes.)

Because of their speed and objectivity photometric methods offer great possibilities for the study of suspensions of bacteria or other cells. The utility of such methods depends largely upon the extent to which the measurements can be interpreted in terms of the suspended cells. This implies an instrument of known optical characteristics, and an understanding of the manner in which such a suspension distributes a light flux incident upon it as a function of

such variables as the size, shape, and numerical concentration of the cells, their index of refraction relative to the suspension medium, and the presence of pigment or of Rayleigh scattering.

There has first been considered the scattering of a collimated beam incident on a suspension of perfect dielectric spheres in a transparent medium. The diameter of the spheres (1 micron) and their refractive index relative to the medium (1.045) have been chosen to resemble as closely as possible a suspension of spherical bacteria in ordinary culture media. It has been shown by Blumer[1] that under these conditions the equations of Mie become useless for calculation of the distribution of the emergent flux.

From probability theory it is shown that the portion of the incident flux transmitted undeviated, without incidence on cells, approximately obeys Beer's law when concentration is represented by the product of the cross-sectional area of a sphere and the number per unit volume. With very large spheres, or high numerical concentrations, systematic departure occurs.

The angular distribution of the flux incident on a single sphere has been calculated by a method similar to that of Shoulejkin,[2] and expressions found to describe the effects on this primary distribution produced by succeeding incidences in a suspension. The fraction of the incident flux emergent within a given solid angle has been computed, as a function of cell size and numerical concentration, for several solid angles, so that the effect of differences in the angular aperture of the photometric system can clearly be seen. The effects of the other variables mentioned above have also been briefly considered with special reference to the design of the photometric system.

[1] H. Blumer, Zeits. f. Physik **38**, 304 (1926).
[2] W. Shoulejkin, Phil. Mag. **48**, 307 (1924).

15. Effect of Ultraviolet Rays on Virulence of Snake Venoms. DAVID I. MACHT, *Pharmacological Research Laboratory, Hynson, Westcott and Dunning, Inc., Baltimore, Md.* (15 minutes.)

Solutions or suspensions of dried scales of venom from cobra, rattlesnake and copperhead were made in physiological saline. The virulence of such suspensions was tested on various animals and on plants. Most of the zoopharmacological experiments were performed on mice injected with venom, while the phytopharmacological tests were made on seedlings of *Lupinus albus* according to the writer's special method.[1] When the potency of a given suspension had been determined, samples thereof in quartz cells were irradiated with mercury vapor lamps for different periods of time. It was found that exposure for ten minutes to the rays of a Kromayer lamp produced appreciable deterioration of the venom, and longer exposures rapidly effected more detoxification. Rays from a Kromayer lamp, passing through a Wood filter, produced little deterioration even after long exposure of venom. Radiations from a mercury lamp, passing through a chlorine gas filter, were more effective than the longer ultraviolet rays. Cobra venom is more resistant to the influence of ultraviolet rays than Crotalus venom. A different series of experiments was made on mice first injected with a potent snake venom and afterwards exposed to ultraviolet rays for possible therapeutic action. No antidotal effect was noted; in fact, these animals succumbed more rapidly than the controls injected with the same venom but not irradiated. Further experiments were made with suspensions of venom treated with x-rays and radium emanations, respectively. The results of this investigation, together with other interesting physiological data obtained from experiments on larger animals, will be described in a complete paper. While the ultraviolet rays destroy snake venom in quartz vessels, they are useless for therapeutic irradiation of poisoned animals except when directly applied to open wounds containing the venom.

[1] Macht, Science **71**, 302 (1930).

16. The Measurement of Gloss. RICHARD S. HUNTER, *Henry A. Gardner Laboratory, Washington, D. C.* (15 minutes.)

This paper is a discussion of the general problem of gloss measurement. Glossiness and luster are effects to be associated with regular reflection. However, no single measurement of reflection seems adequate to evaluate gloss under all the circumstances in which it appears. The different devices which are offered as gloss measuring instruments can be presented as evidence of the number of types of reflection measurements that are called gloss measurements. Furthermore, the different viewpoints from which gloss is evaluated seem to demand these different methods of measurement.

Some examples of these different types of gloss evaluation are to be shown. The instruments used to measure gloss can be compared to show the physical methods employed. Two general methods of gloss measurement are used: (1) measurement of the resolution, or sharpness of images reflected in the gloss surface and (2) measurement of the regular reflection coefficient of the gloss surface. Because, in general, each particular type of instrument operates under its own set of conditions, no two instruments measure the same thing.

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Detroit Paint Production Club Standard Method of Gloss Measurement, Scientific Section Circular No. 423 (1933) of the National Paint, Varnish and Lacquer Association, Washington, D. C.
R. S. Hunter, Scientific Section Circular No. 456 (1934) of the National Paint, Varnish and Lacquer Association, Washington, D. C.
R. L. Ingersoll, J. Opt. Soc. Am. **5**, 213 (1921).
L. A. Jones, J. Opt. Soc. Am. **6**, 140 (1922).
M. O. Pelton, Trans. Opt. Soc. (London) **31**, 184 (1929–30).
A. H. Pfund, Proc. A. S. T. M. **25** (II), 396 (1925); J. Opt. Soc. Am. **20**, 23 (1930).
Sward and Levy, Scientific Section Circular No. 380 (1931) of the National Paint, Varnish and Lacquer Association, Washington, D. C.

17. The Interpretation of Data Obtained with Spectrophotometers of the Polarization Type. ROBERT D. NUTTING, *Massachusetts Institute of Technology.* (15 minutes.)

Many spectrophotometers utilize polarizing prisms and determine the reflectance of the sample by plane polarized light. In general, the reflectance of the sample is independent of the azimuth of the plane of polarization but there are some materials, notably textiles, for which the reflectance depends upon the azimuth. It can be shown mathematically that, if the reflectance is measured for two azi-

muths at right angles to one another, the arithmetic average of the two values is the reflectance for ordinary, or unpolarized, light. This is true regardless of the azimuths selected. Experimental data are presented which indicate the applicability of this principle to the measurement of the color of textile fabrics.

18. The Eye as an Integrator of Short Light Flashes. J. W. BEAMS, *University of Virginia.* (10 minutes.)

The way in which the eye integrates the intensity of a series of very short flashes of light has been investigated. Light from a straight incandescent filament, after reflection in a rotating mirror, was brought to focus on a slit. The duration of each flash was equal to the time required for the image of the filament to sweep across the slit and the number of flashes per second was equal to the rotational speed of the mirror. Since the number of flashes per second was always great enough to avoid flicker, their integrated intensity could be compared with a steady continuous source. For this comparison source, light from the incandescent filament mentioned above was passed through two nicol prisms and brought to focus on the upper part of the slit. In the field of view of an eyepiece placed behind the slit the image of this comparison source appeared above and just touching that consisting of the light flashes. It was found when the intensity of the comparison source was adjusted, by rotating one nicol, to give a "match" of the two images, that this "match" was independent of the rotational speed of the mirror over the range studied. The rotating mirror[1] was both supported and driven by air jets and its axis of rotation was accurately vertical at all rotational speeds used. Since the amount of energy that passed through the slit was independent of the rotational speed of the mirror but the duration of each flash was inversely proportional to this speed, it is concluded that over the range studied the eye integrates the energy in the light flashes in the same way and independently of the length of the light flashes. The duration of the light flashes was varied from 10^{-6} to 10^{-3} second and the two images in the field of view of the observing eye piece could be "matched" with a precision of 2.5 percent.

Bibliography:
[1] See Beams, R. S. I. 1, 667 (1930); Beams, Weed and Pickels, Science **78**, 338 (1933).

19. A General Purpose Amplifier and Its Specific Use in the Actinometry of Photoflash Lamps. S. MCKAY GRAY AND HOWARD J. ECKWEILER, *Electrical Testing Laboratories, New York, N. Y.* (20 minutes.)

In consequence of a need for the amplification of both transient and audiofrequency periodic electrical phenomena sufficient to operate a Duddell-type oscillograph with an element sensitivity of 1.2 mm/m.a., a direct-coupled vacuum-tube amplifier has been constructed incorporating the following features: A transconductance of 0.44 ampere/volt which is constant over a range of 0.12 ampere; a voltage-amplification of 578; a maximum *linear* current amplification of 4.4×10^7 requiring an input resistance of 100 megohms; the compensation of the interelectrode capacitances to effect the cancellation of distortion introduced by the shunted currents necessary to charge these capacitances; and a maximum of stability.

The specific use of this general purpose amplifier in the measurement of the actinic intensity *vs.* time function of photoflash lamps and flashlight powders is described as an illustrative example of its employment.

A brief outline is given of the method employed for the correction of the spectral sensitivity of a photoelectric cell to approximate that of Eastman *Verichrome* or of Eastman *Super Pan* film. The methods of standardization, etc., are either detailed or outlined.

Extension into other fields of usefulness is indicated by reference to measurement of transient light phenomena, the determination of rapid color temperature variations, the calibration of photographic shutters, etc.

The present paper will appear in full in Review of Scientific Instruments.

20. Relative Photographic Effectiveness of Some Light Sources. W. E. FORSYTHE AND M. A. EASLEY, *Incandescent Lamp Department, General Electric Company, Nela Park, Cleveland, Ohio.* (20 minutes.)

Besides the regular incandescent lamps and the blue bulb photographic lamp there are two photo-flood lamps and a movie-flood lamp, and in addition four photo-flash lamps that are used for taking pictures under different conditions. Some of the characteristic data of these lamps are given in Table I.

The relative photographic effectiveness of the photo-flash lamps was measured for both the maximum intensity and total light output while the incandescent lamps were measured at normal intensity. The regular 115 volt, 500 watt lamp was used as a standard, to calibrate the film with 1/500 second exposure time for the maximum intensity of the photo-flash lamp and 1/25 second for all other calibrations. The time of exposure for the incandescent lamps was 1/25 second and for the photo-flash lamps was determined by the length of flash.

The films used for measuring the relative photographic effectiveness were developed in Eastman D76 developer to a contrast of approximately 0.8 and the values of the relative effectiveness given in the table are for a density of unity. The relative photographic effectiveness of these sources differs with the type of emulsion. The results given in the table are the average values obtained on two different brands of commercial, orthochromatic and supersensitive panchromatic films.

TABLE I.

Lamp	Volts	Lumen output	Color temp.	Photographic effectiveness Total 1	Total 2	Total 3	Maximum 1	Maximum 2	Maximum 3
500-W	115	9800	2950°K	1	1	1	1	1	1
1500-W (blue bulb)	115	18000		3.7	3.1	2.6			
250-W photo-flood	115	8600	3490	1.3	1.2	1.0			
1000-W "	115	32000	3410	4.6	4.5	3.9			
2000-W movie-flood	115	67000	3430	11.2	10.8	9.4			
Photo-flash	Output in lumen seconds								
No. 10		26000		210	142	110	500	460	330
No. 20		52000		310	260	245	1180	840	580
No. 75		180000		1250	900	750	1580	1380	1050

Notes: 1=Commercial film
2=Orthochromatic film
3=SS panchromatic film

The present paper will appear in full in J. O. S. A.

21. Simplified Microdensitometer Based on that of Hartmann. S. JACOBSOHN AND W. H. KLIEVER, *The Gaertner Scientific Corporation.* (5 minutes.)

The microphotometer of Hartmann suffers from several disadvantages which have been removed in this construction. The stage has been inclined to the horizontal and the reflection at the photometer cube transferred to the element which images the wedge. A convenient posture is thus attainable and the annoyance of adjusting the plate in the field by the help of a perverted image is avoided. The stage is provided with a T-square rest for the plate, adjustable by rack and pinion. The field of spectrum lines may be seen at the same time as measurements are made owing to a construction of the photometer cube having a slit of rather short length. Instead of the photographic wedge of the original instrument a neutral gelatine wedge is employed and grain introduced by a fixed plate, on which the comparison microscope is focused. The wedge readings are therefore proportional to the densities. Data are given showing the sensitivity and accuracy of the instrument.

Bibliography:
Zeits. f. Instkde. **19**, 77.
The present paper will appear in full in R. S. I.